Hinkkanen, Aimo; **Martin, Gaven J.**; **Mayer, Volker**
Local dynamics of uniformly quasiregular mappings. (English) Zbl 1067.30043
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A quasiregular map $f\colon \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}^n$ is a quasiconformal map without the injectivity property. Such a map is called uniformly quasiregular if all the iterates f^k have the same distortion bound. The Fatou set and Julia set of f are defined as in the classical Fatou-Julia theory of rational maps on $\overline{\mathbb{C}}$. The object of this paper is to study the local dynamics of the sequence of iterates f^k. As in the classical case the fixed points of f play an essential role.

If a map $f\colon \overline{\mathbb{R}}^n \to \overline{\mathbb{R}}^n$ is differentiable at some point $x_0 \in \overline{\mathbb{R}}^n$, then the derivative is a linear map $L_{x_0}\colon \mathbb{R}^n \to \mathbb{R}^n$. But a quasiregular map need not be differentiable. If x_0 is a fixed point of f which is not a branch point of f, the authors define a so-called generalized derivative L_{x_0} which is a uniformly quasiconformal map. Using the classification of uniformly quasiconformal maps (they are either loxodromic, elliptic or parabolic) they get an analytic classification of the different types of fixed points which generalizes in a natural way the usual one of holomorphic functions. As an application they get that uniformly quasiregular maps do have precisely the same type of Fatou components as rational functions.

Examples of uniformly quasiregular maps with attracting, super-attracting or repelling fixed points are known from the papers of *T. Iwaniec* and *G. Martin* [Ann. Acad. Sci. Fenn., Math. 21, 241–254 (1996 Zbl 0860.30019)] and of *V. Mayer* [Conform. Geom. Dyn. 1, 104–111 (1997 Zbl 0897.30008)]. Here, the authors construct new examples with parabolic dynamics, and they also show that such a map need not admit a quasiconformal linearization in its attracting parabolic petal. Finally, they show that the natural candidates for a linearization are not affine maps but the generalized derivatives mentioned above. In fact, they prove that a uniformly quasiregular map f with distortion bound K can always be K-quasiconformally conjugated near an attracting or repelling fixed point to a generalized derivative.

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MSC:

30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations
37F10 Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets
37F30 Quasiconformal methods and Teichmüller theory, etc. (dynamical systems) (MSC2010)
37F50 Small divisors, rotation domains and linearization in holomorphic dynamics

Keywords:

quasiregular map; iteration; Fatou set; Julia set; fixed point; generalized derivative

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