Kremer, Philip; **Mints, Grigori**
Dynamic topological logic. (English) Zbl 1067.03028
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Three important research areas meet together in the abstract framework of dynamic topological logic (DTL): the topological semantics for S4, topological dynamics, and temporal logic. It is known that S4 can be understood as the logic of topological spaces, and $\Box$ can be understood as a topological modality (with the meaning of the topological interior). Thus, the topological semantics for S4 is based on topological spaces rather than Kripke frames. On the other hand, topological dynamics studies the asymptotic properties of continuous maps on topological spaces. A dynamic topological system is a topological space X together with a continuous function f which can be thought of in temporal terms as moving the points of the topological space.

Dynamic topological logics are the logics of dynamic topological systems, just as S4 is the logic of topological spaces, and are defined for a trimodal language with an S4-ish topological modality, and two temporal modalities ('next' and 'henceforth') both interpreted using the continuous function f. In particular, 'next' expresses f's action on X from one moment to the next, and 'henceforth' expresses the asymptotic behaviour of f.

The authors introduce the dynamic topological analogues of Kripke models, the dynamic Alexandrov models, in order to set a precise definition of dynamic topological model; then, a semantic definition of the dynamic topological logic generated by a class $\mathcal{T}$ of topological spaces and/or a class $\mathcal{F}$ of continuous functions. Later, several specific DTLs are considered, presenting their properties and axiomatizing some of their next-interior fragments. Finally, a sound and complete axiomatization of a DTL is given in a particular trimodal fragment of the language in which the temporal modalities cannot occur in the scope of a topological modality.

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MSC:

03B45 Modal logic (including the logic of norms)
03B44 Temporal logic
54H20 Topological dynamics (MSC2010)

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