Coates, J.; **Schneider, P.**; **Sujatha, R.**
Modules over Iwasawa algebras. (English) Zbl 1061.11060
J. Inst. Math. Jussieu 2, No. 1, 73-108 (2003).

Let p be a prime number, and G a compact p-adic Lie group. The Iwasawa algebra of G is defined by

$$\Lambda(G) := \varprojlim_H \mathbb{Z}_p[G/H],$$

where H runs over the set of all open normal subgroups of G and $\mathbb{Z}_p$ is the ring of p-adic integers. There exist several important examples of finitely generated modules over $\Lambda(G)$ appearing in arithmetic geometry; for instance, some examples come from elliptic curves without complex multiplication. Such $\Lambda(G)$-modules are a natural generalization of Iwasawa theory.

When $G = \mathbb{Z}_p^d$, d a natural number, the structure of finitely $\Lambda(G)$-modules is known, up to pseudo-isomorphism. The structure theorem is due to *K. Iwasawa* [Bull. Am. Math. Soc. 65, 183–226 (1959; Zbl 0089.02402)] and *J.-P. Serre* [Sémin. Bourbaki 11 (1958/59), No. 174 (1959; Zbl 0119.27603)].

The aim of the paper under review is to extend this structure theorem to the noncommutative case. Let A be a noncommutative ring with unit. The authors define pseudo-null A-modules which is a generalization of the definition of pseudo-null $\Lambda(G)$-modules given by *O. Venjakob* in [J. Eur. Math. Soc. (JEMS) 4, No. 3, 271–311 (2002; Zbl 1049.16016)]. The structure theorem obtained is the following: "Let G be a p-valued compact p-adic Lie group, and let M be a finitely generated torsion $\Lambda(G)$-module. Let M_0 be the maximal pseudo-null submodule of M. Then there exist non-zero left ideals $L_1, \ldots, L_m$ and a $\Lambda(G)$-monomorphism

$$\phi : \bigoplus_{i=1}^{m} \Lambda(G)/L_i \to M/M_0$$

with $\operatorname{coker}\phi$ pseudo-null. Furthermore, all left ideals $L_1, \ldots, L_m$ having this property are reflexive as $\Lambda(G)$-modules."

The results about structure theorems for any finitely generated torsion A-modules are obtained by using two approaches. One is inspired by Venjakob's work which treated the case of finitely generated $\Lambda(G)$-modules annihilated by some power of p. The authors take advantage of a well-known filtration on $\Lambda(G)$ (using techniques from microlocalization) and extend Bourbaki's proof of the structure theory in the commutative case to finitely generated torsion modules over $\Lambda(G)$, for a wide class of p-adic Lie groups G. The second approach is following work of *M. Chamarie* [Lect. Notes Math. 1029, 283–310 (1983; Zbl 0528.16003)] on modules over maximal orders. For modules M such that M/M_0 has non-zero global annihilator, the authors exploit Chamarie's methods to prove some kind of uniqueness for the ideals $L_1, \ldots, L_m$ and to define the notion of the characteristic ideal of M.

In the last section some examples are given. The Pontryagin dual of the Selmer group of an elliptic curve E without complex multiplication over a number field F with $G = \operatorname{Gal}(F_\infty/F)$ where $F_\infty = F\big(\bigcup_{n=1}^{\infty} E_{p^n}\big)$, E_{p^n}, the group of p^n-division points on E ($p \geq 5$), is an important $\Lambda(G)$-module in arithmetic algebraic geometry. In the case of elliptic curves with complex multiplication the structure of this $\Lambda(G)$-module is well known. The authors compute examples for some elliptic curves without complex multiplication as an application of the structure theorem.

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MSC:

11R23	Iwasawa theory
11G05	Elliptic curves over global fields
16D70	Structure and classification for modules, bimodules and ideals (except as in 16Gxx), direct sum decomposition and cancellation in associative algebras)
22E50	Representations of Lie and linear algebraic groups over local fields

Keywords:

Iwasawa theory; Iwasawa algebras; elliptic curves; representation of p-adic Lie groups

Full Text: DOI