Borisov, Lev A.; **Mavlyutov, Anvar R.**

String cohomology of Calabi-Yau hypersurfaces via mirror symmetry. Appendix: *G*-polynomials. (English) Zbl 1055.14044

Adv. Math. 180, No. 1, 355-390 (2003).

The paper under review proposes a construction of string cohomology spaces for Calabi-Yau hypersurfaces that arise in the Batyrev mirror symmetry construction [cf. *V. Batyrev*, J. Algebr. Geom. 3, No. 3, 493–535 (1994; Zbl 0829.14023)], with spaces defined explicitly in terms of the corresponding reflexive polyhedra. The construction starts with the description of the cohomology of semiample hypersurfaces in toric varieties and then uses mirror symmetry to provide a conjectural string cohomology of Calabi-Yau hypersurfaces. This construction gives the correct (bigraded) dimension of the space, and further, a finite-dimensional family of string cohomology spaces rather than just a single-string cohomology space. That is, string cohomology space depends not only on the complex structure (the defining polynomial f), but also on some extra parameter ω. For special values of this parameter of an orbifold Calabi-Yau hypersurface, the construction gives the orbifold Dolbeault cohomology, recovering the result of *W. Chen* and *Y. Ruan* [Commun. Math. Phys. 248, 1–31 (2004; Zbl 1063.53091)]. However, for non-orbifold Calabi-Yau hypersurfaces, there is no natural choice for ω, and this implies the dependence of the general definition of string cohomology space on some parameter. In case of Calabi-Yau hypersurfaces, this particular parameter ω corresponds to the defining polynomial of the mirror Calabi-Yau hypersurface. In general, this parameter should be related to the "string complexified Kähler class" which is yet to be defined. An attempt is made to extend the definition of string cohomology space beyond the Calabi-Yau hypersurfaces. A conjectural definition of string cohomology vector spaces is presented for stratified varieties with $\mathbb{Q}$-Gorenstein toroidal singularities that satisfy certain restrictions on the types of singular strata. This definition would involve intersection cohomology of the closures of strata as well as perverse sheaves. This conjectural definition gives the correct bigraded dimension and also reproduces orbifold cohomology of a $\mathbb{Q}$-Gorenstein toric variety as a special case.

Reviewer: Noriko Yui (Kingston)

MSC:

14J32	Calabi-Yau manifolds (algebro-geometric aspects)
14M25	Toric varieties, Newton polyhedra, Okounkov bodies
81T30	String and superstring theories; other extended objects (e.g., branes) in quantum field theory
32Q25	Calabi-Yau theory (complex-analytic aspects)
14F43	Other algebro-geometric (co)homologies (e.g., intersection, equivariant, Lawson, Deligne (co)homologies)

Cited in **3** Reviews
Cited in **19** Documents

Keywords:

toric varieties; intersection cohomology; orbifold Dolbeault cohomology

Full Text: DOI arXiv

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