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The uniqueness of tangent cones for Yang-Mills connections with isolated singularities.
(English) Zbl 1049.53021
Adv. Math. 180, No. 2, 648-691 (2003).

Let M be an n-dimensional manifold and E a vector bundle associated to a principal bundle P on M with compact structure group G. Assuming that (M, g) is an n-dimensional Riemannian manifold, $n \geq 5$, $x_0 \in M$ and (E, h) a bundle over M-$\{x_0\}$, h being a G-invariant metric, let A be a Yang-Mills connection on E with an isolated singularity at x_0. The author proves the uniqueness, up to gauge transformations, of the tangent cone (or tangent connection) of A at x_0, under a quadratic growth assumption on the curvature of A in a neighbourhood of x_0.

He also gives an estimate of the rate of the asymptotic convergence of A to its cone when A is stationary, since in this case the a priori estimates of *K. K. Uhlenbeck* [Commun. Math. Phys. 83, 31–42 (1982; Zbl 0499.58019)] and *H. Nakajima* [J. Math. Soc. Japan 40, No. 3, 383–392 (1988; Zbl 0647.53030)] ensure the growth condition on the curvature of A. Furthermore, such a convergence rate becomes faster when the tangent cone is assumed to be integrable, even for non-stationary A.

Technically, since the Yang-Mills equation has a degenerate elliptic nature, the author looks for a suitable gauge (using Coulomb gauge) having a long time existence so that he can adapt *L. Simon's* method [Ann. Math. (2) 118, 525–571 (1983; Zbl 0549.35071)]. This is achieved by means of a partition of the existence interval of the solution in three parts corresponding to different growth behaviours of the norm of the solution. Such behaviours are modelled on those of the solutions to the linearized equations, and this allows to control their norms on each interval using techniques differing from the variational inequality by Simon and the property that, under the constructed gauge, the time derivative of the connection is uniformly small on the existence interval.

Finally, the author considers Yang-Mills (Y-M) flows which start from a connection sufficiently close, in norm, to a smooth local minimizer of the Y-M functional and he proves the asymptotic convergence to a suitable Y-M connection near the minimizer. Again he constructs a suitable gauge and uses the Simon result for parabolic evolution equations.

Reviewer: Anna Maria Pastore (Bari)

MSC:

53C07	Special connections and metrics on vector bundles (Hermite-Einstein, Yang-Mills)
58E15	Variational problems concerning extremal problems in several variables; Yang-Mills functionals

Cited in **8** Documents

Keywords:

Yang-Mills connection; gauge transformations; tangent cone; asymptotic convergence; variational inequality

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