Hosono, Shinobu

Counting BPS states via holomorphic anomaly equations. (English) Zbl 1046.81086

Yui, Noriko (ed.) et al., Calabi-Yau varieties and mirror symmetry. Providence, RI: American Mathematical Society (AMS) (ISBN 0-8218-3355-3/hbk). Fields Inst. Commun. 38, 57-86 (2003).

Let S be a surface obtained by blowing up 9 base points of 2 generic cubics in P^2. The author studies the Gromov-Witten invariants $N_g(\beta)$ with $\beta \in H_2(S,Z)$ of a rational elliptic surface S using holomorphic anomaly equation (HA eq.). Let F, σ in $H^2(S,Z)$ be the fiber class and the class of a section of the elliptic fibration: $S \to P^1$. From $N_g(d,n) := \sum_{\beta.\sigma=d, \beta.F=n} N_g(\beta)$, $Z_{g;n}(q) := \sum_{d\geq 0} N_g(d,n)q^d = P_{g,n}(E_2(q), E_4(q), E_6(q))q^{n/2}/\eta(q)^{12n}$ is given. E_2, E_4, E_6 are Eisenstein series, $\eta(q) = q^{1/24}\prod_{m>0}(1-q^m)$. He treats the HA eq.:

$$\partial Z_{g;n}/\partial E_2 = 24^{-1} \sum_{g'+g''=g} \sum_{s=1\sim n-1} S(n-s) Z_{g';s^*} Z_{g'';n-s} + n(n+1)Z_{g-1;n}/24,$$

with the initial data $Z_{0;1} = q^{1/2}E_4(q)/\eta(q)^{12}$. Using the affine E_8 symmetry which arises as isomorphisms of rational elliptic surfaces, he determines $N_g(\beta)$ with $(\beta, F) = n = 1, 2, 3, 4$ and genus $g = 2^{-1}\{(\beta,\beta) - (\beta, F) + 2\} \leq 10$. The conjectured numbers $n_g(\beta)$ of BPS states with spin g and charge β are obtained from $N_g(\beta)$. A conjecture relating to the ambiguity of $F_g(p,q) := \sum_{n\geq 1} Z_{g;n}p^n$ $(g \geq 2)$ is also given.

For the entire collection see [Zbl 1022.00014].

Reviewer: Hideo Yamagata (Osaka)

MSC:

81T30	String and superstring theories; other extended objects (e.g., branes) in quantum field theory
14N35	Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)
53D45	Gromov-Witten invariants, quantum cohomology, Frobenius manifolds
14J81	Relationships between surfaces, higher-dimensional varieties, and physics
14J32	Calabi-Yau manifolds (algebro-geometric aspects)
14J27	Elliptic surfaces, elliptic or Calabi-Yau fibrations
11F23	Relations with algebraic geometry and topology

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