Esteva, Francesc; **Godo, Lluís**; **Hájek, Petr**; **Montagna, Franco**
Hoops and fuzzy logic. (English) Zbl 1039.03016
J. Log. Comput. 13, No. 4, 531-555 (2003).

BL-algebras are the algebras of Hájek's basic logic [see *P. Hájek*, Metamathematics of fuzzy logic. Trends in Logic – Studia Logica Library 4, Kluwer Academic Publishers, Dordrecht (1998; Zbl 0937.03030)]. Working in the neighbourhood of BL-algebras and their 0-free subreducts, the authors investigate the positive fragment of various logics related to Basic Logic. All the resulting structures are particular cases of Büchi-Owens hoops. Just as Basic Logic is the logic of continuous t-norms, MTL is the logic of left-continuous t-norms. The authors also investigate the weaker structures arising from MTL, called semi-hoops. They provide axiomatizations and prove completeness and conservativeness results for several such logics. Turning attention to predicate logics, the authors prove further completeness results with respect to safe interpretations over the corresponding totally ordered structures, generalizing previous results of Hájek for Basic Logic. Mutual interpretability issues are dealt with in a final section. Since all interpretations turn out to be computable in polynomial time, various co-NP-completeness results for the tautology problem of these logics are obtained from the well-known corresponding results about Łukasiewicz logic [*D. Mundici*, "Satisfiability in many-valued sentential logic is NP-complete", Theor. Comput. Sci. 52, 145–153 (1987; Zbl 0639.03042)] and about Basic Logic etc. [*M. Baaz*, *P. Hájek*, *F. Montagna*, and *H. Veith*, "Complexity of t-tautologies", Ann. Pure Appl. Logic 113, 3–11 (2002; Zbl 1006.03022)].

Reviewer: Daniele Mundici (Firenze)

MSC:

03B52	Fuzzy logic; logic of vagueness
06D35	MV-algebras
03D15	Complexity of computation (including implicit computational complexity)
68Q17	Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Cited in **50** Documents

Keywords:

implicational fragment; basic logic; MTL t-norm; BL-algebra hoop; MV-algebra

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