Székely, Gábor J.; **Bakirov, Nail K.**

Extremal probabilities for Gaussian quadratic forms. (English) Zbl 1031.60018

Probab. Theory Relat. Fields 126, No. 2, 184-202 (2003).

Denote by Q an arbitrary positive semidefinite quadratic form in centered Gaussian random variables such that $E(Q) = 1$. The main result of the paper is the following statement: $\inf_Q P(Q \leq x) = P(\chi_n^2/n \leq x), x > 0$, where χ_n^2 is chi-square distributed random variable with $n = n(x)$ degrees of freedom, $n(x)$ is a non-increasing function of x. Moreover, it is proved that $n = 1$ iff $x > x(1)$, $n = 2$ iff $x \in [x(2), x(1)]$, etc, $n(x) \leq \operatorname{rank}(Q)$, where $x(1) = 1.5364\cdots, x(2) = 1.2989\cdots, \dots$. It is noted that a similar statement is not true for the supremum: if $1 < x < 2$ and Z_1, Z_2 are independent standard Gaussian random variables, then $\sup_{0\leq\lambda\leq 1/2} P\{\lambda Z_1^2 + (1-\lambda)Z_2^2 \leq x\}$ is taken not at $\lambda = 0$ or at $\lambda = 1/2$ but at $0 < \lambda < \lambda(x) < 1/2$, where $\lambda(x)$ is a continuous, increasing function from $\lambda(1) = 0$ to $\lambda(2) = 1/2$, e.g. $\lambda(1.5) = .15\cdots$. Applications of the results include asymptotic quantiles of U- and V-statistics, signal detection, and stochastic orderings of integrals of squared Gaussian processes.

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MSC:

60E15 Inequalities; stochastic orderings
60G15 Gaussian processes
62G10 Nonparametric hypothesis testing

Keywords:

Gaussian random vector; Gaussian quadratic form; chi-square distributed random variable

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