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The fixed point property in modal logic. (English) Zbl 1031.03039

Notre Dame J. Formal Logic 42, No. 2, 65-86 (2001).

Summary: This paper deals with the modal logics associated with (possibly nonstandard) provability predicates of Peano Arithmetic. One of our goals is to present some modal systems having the fixed point property and not extending the Gödel-Löb system **GL**. We prove that, for every $n \geq 2$, $K + \Box(\Box^{n-1}p \to p) \to \Box p$ has the explicit fixed point property. Our main result states that every complete modal logic L having Craig's interpolation property and such that $L \vdash \Delta(\nabla(p) \to p) \to \Delta(p)$, where $\nabla(p)$ and $\Delta(p)$ are suitable modal formulas, has the explicit fixed point property.

MSC:

03B45 Modal logic (including the logic of norms)
03F45 Provability logics and related algebras (e.g., diagonalizable algebras)
03F40 Gödel numberings and issues of incompleteness
03F30 First-order arithmetic and fragments

Keywords:

provability predicates of Peano Arithmetic; fixed point property; modal logic

Full Text: DOI

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