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Investigation of a two-dimensional spectral element method for Helmholtz's equation. (English) Zbl 1024.65112

J. Comput. Phys. 189, No. 1, 111-129 (2003).

Summary: A spectral element method is developed for solving the two-dimensional Helmholtz equation, which is the equation governing time-harmonic acoustic waves. Computational cost for solving Helmholtz equation with the Galerkin finite element method increases as the wave number increases, due to the pollution effect. Therefore a more efficient numerical method is sought.

The comparison between a spectral element method and a second-order finite element method shows that the spectral element method leads to fewer grid points per wavelength and less computational cost, for the same accuracy. It also offers the same advantage as the finite element method to address complex geometry and general material property.

Some simple examples are addressed and compared with the exact solutions to confirm the accuracy of the method. For unbounded problems, the symmetric perfectly matched layer (PML) method is applied to treat the non-reflecting boundary conditions. In the PML method, a fictitious absorbing layer is introduced outside the truncated boundary.

MSC:

65N35 Spectral, collocation and related methods for boundary value problems involving PDEs

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation

Cited in **16** Documents

Keywords:

numerical examples; comparison of methods; spectral element method; Helmholtz equation; acoustic waves; Galerkin finite element method; unbounded problems; symmetric perfectly matched layer method; nonreflecting boundary conditions

Software:

LASPack

Full Text: DOI

References:

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