Schindler, Ralf-Dieter; **Steel, John**; **Zeman, Martin**
Deconstructing inner model theory. (English) Zbl 1017.03030
J. Symb. Log. 67, No. 2, 721-736 (2002).

The authors are not "deconstructing" inner model theory in the paper under review, but rather reconstructing (Mitchell-Steel type) inner model theory after Jensen and Zeman discovered problems with it.

Inner model theory for models with Woodin cardinals was developed by *W. J. Mitchell* and *J. R. Steel* [Fine structure and iteration trees, Lecture Notes in Logic 3. Berlin: Springer-Verlag (1994; Zbl 0805.03042)] ("FSIT" in the following) and *J. R. Steel* [The core model iterability problem, Lecture Notes in Logic 8. Berlin: Springer-Verlag (1996; Zbl 0864.03035)]. One of the key properties of the Mitchell-Steel fine structure was the Initial Segment Condition (ISC) of extender sequences, and many of the proofs in the Mitchell-Steel setup depend on the ISC. While working on the proof of the combinatorial principle "square" in inner models [*E. Schimmerling* and *M. Zeman*, Bull. Symb. Log. 7, 305-314 (2001; Zbl 0992.03062)], Zeman realized that a particular kind of extenders (called "type Z extenders" in the paper under review) causes trouble for the ISC: an extender sequence indexed as in FSIT satisfying the ISC cannot have type Z extenders, but on the other hand any moderately rich extender sequence must have extenders with type Z initial segments (Theorem 2.3). In the paper under review, the authors define a revised ISC (Definition 2.4) and fix the proofs of FSIT.

Another problem is the possible occurrence of anomalous phalanxes (discovered by Jensen) in the proof of the Solidity Lemma (Theorem 8.1 of FSIT). This problem is dealt with in §3 of the paper under review.

In §4, the authors fix a gap in the proof of Dodd solidity due to Steel [*E. Schimmerling*, Ann. Pure Appl. Logic 74, 153-201 (1995; Zbl 0834.03018)].

R. B. Jensen developed a different fine structure for inner models with Woodin cardinals ["A new fine structure for higher core models", circulated handwritten notes (1997)] that uses a different method of indexing the extenders. The Jensen indexing avoids the problems that are solved in this paper.

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MSC:

03E45 Inner models, including constructibility, ordinal definability, and core models
03E55 Large cardinals

Cited in **17** Documents

Keywords:

initial segment condition; fine structure; type Z extenders; Mitchell-Steel models; inner model theory

Full Text: DOI

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[1] The core model iterability problem 8 (1996) · Zbl 0864.03035
[2] Fine structure and iteration trees 3 (1994) · Zbl 0805.03042
[3] A weak Dodd-Jensen lemma 64 pp 1285– (1999)
[4] DOI: 10.1016/0168-0072(94)00036-3 · Zbl 0834.03018 · doi:10.1016/0168-0072(94)00036-3