Byrne, Helen; **Matthews, Paul**

Asymmetric growth of models of avascular solid tumours: exploiting symmetries. (English)

Zbl 1014.92019

IMA J. Math. Appl. Med. Biol. 19, No. 1, 1-29 (2002).

Summary: A mathematical model of avascular tumour growth is studied. Attention focuses on the stability of radially symmetric model solutions to perturbations involving spherical harmonics $Y_{lm}(\theta, \varphi)$. Linear theory is used to identify bifurcation points at which the radially symmetric steady state loses stability. The first modes to become unstable are shown to correspond to the $l = 2$ spherical harmonics. Results from group theory and weakly nonlinear analysis indicate that the structure of the $l = 2$ bifurcation point is a transcritical bifurcation in which all nontrivial solution branches are unstable.

By proceeding to third-order and focusing on a special set of parameter values for which the quadratic terms are negligible, it is shown that the system's behaviour in a neighbourhood of the $l = 2$ bifurcation point is governed by a subcritical bifurcation. In consequence, the nontrivial asymmetric solution branches in a neighbourhood of the bifurcation point are unstable. The branches of asymmetric solutions bound the domain of attraction of the radially symmetric tumour configuration where it is locally stable. The analytical results that are derived lead us to conjecture that any stable asymmetric tumour structures will involve spherical harmonics of order $l \geq 3$.

MSC:

92C50	Medical applications (general)
34C60	Qualitative investigation and simulation of ordinary differential equation models
34C23	Bifurcation theory for ordinary differential equations

Keywords:

tumour growth; transcritical bifurcation; weakly nonlinear analysis; group theory