Rubinov, A. M.; **Vladimirov, A. A.**

Differences of convex compacta and metric spaces of convex compacta with applications: A survey. (English) Zbl 1002.49022

Demyanov, V. (ed.) et al., Quasidifferentiability and related topics. Dedicated to Prof. Franco Giannessi on his 65th birthday and to Prof. Diethard Pallaschke on his 60th birthday. Dordrecht: Kluwer Academic Publishers. Nonconvex Optim. Appl. 43, 263-296 (2000).

The authors give a nice survey about the different definitions of algebraic set differences. It is pointed out that the most definitions base essentially on the Minkowski duality, i.e., on the (bijective) correspondence φ between the family Y_n of all convex compact sets of $\mathbb{R}^n$ and the family P_n of all sublinear functions $p:\mathbb{R}^n\to\mathbb{R}$.

Let $A,B\in Y_n$ and $p_A,p_B\in P_n$ be the associated support functions. Since the difference p_A-p_B is not convex in general, a set difference between A and B can be expressed by

$$A\ominus B=\varphi^{-1}(C(p_A-p_B)),$$

where $C(q)$ is a suitable convexification of the positively homogeneous function $q:\mathbb{R}^n\to\mathbb{R}$. So the special set differences depend on the choice of the special convexification of p_A-p_B.

In the first part of the paper, some well-known set differences are discussed and compared, especially:

$*$ the difference using the metric projection $C(q)=\mathrm{Pr}_\infty(q)$ which is the solution of the optimization problem

$$\max\{q(x)-p(x)\mid \|x\|\le 1\}\to\min,\quad p\in P_n.$$

$*$ the $*$-difference where $C(q)$ is the greatest convex minorant of q. By this we get the well-known representation

$$A\ominus B=\{x\mid B+x\subset A\}.$$

$*$ the Demyanov difference where $C(q)$ is the Clarke upper derivative of q. Here we have the representation by the Clarke subdifferential according to

$$A\ominus B=\partial_{Cl}(p_A-p_B)(0).$$

$*$ the exposed difference defined by

$$A\ominus B=\{\nabla p_A(u)-\nabla p_B(u)\mid u\in T_A-T_B\},$$

where T_A and T_B are the sets of points where p_A and p_B are differentiable, i.e., where the faces $A(u)$ and $B(u)$ are singletons (hence exposed points).

$*$ the quasidifferential according to

$$A\ominus B=\mathrm{cl}\ \mathrm{co}\bigcup\{A(u)-B(u)\mid u\in\mathbb{R}^n,u\ne 0\}.$$

In the second part of the paper, the authors present some applications of set-differences in nonsmooth analysis, especially regarding the representation of generalized subdifferentials of DC and quasidifferentiable functions, the approximation of linear set-valued mappings and the construction of suitable metrics (the Demyanov metric and the Bartels-Pallaschke metric) in the space Y_n. Convergence, continuity and differentiability properties of polyhedral-valued mappings with respect to these metrics in comparison with the Hausdorff metric are pointed out.

For the entire collection see [Zbl 0949.00047].

Reviewer: Jörg Thierfelder (Ilmenau)

MSC:

49J52 Nonsmooth analysis
52A20 Convex sets in n dimensions (including convex hypersurfaces)
58C06 Set-valued and function-space-valued mappings on manifolds
49J53 Set-valued and variational analysis

Keywords:

D-regular sets; algebraic set differences; nonsmooth analysis; generalized subdifferentials; quasidifferentiable functions; set-valued mappings; Demyanov metric; Bartels-Pallaschke metric