**Fukuda, Nobuyuki**
**Inverse and direct images for quantum Weyl algebras.** (English) Zbl 0999.16035
J. Algebra 236, No. 2, 471-501 (2001).

This paper is concerned with quantum analogues of ideas (inverse and direct images, Kashiwara's theorem, and preservation of holonomicity) that are well established for modules over Weyl algebras and are explained in Chapters 14 to 18 of 'A primer of algebraic $\mathcal{D}$-modules' by *S. C. Coutinho* [Lond. Math. Soc. Student Texts 33, Cambridge University Press (1995; Zbl 0848.16019)].

The quantum Weyl algebra $A_n(q, P)$ is generated by $n$ variables and $n$ partial differential operators, or skew partial differential operators, on a quantum affine space. It arises from work of *J. Wess* and *B. Zumino* [Nucl. Phys. B, Proc. Suppl. 18B, 302-312 (1990; Zbl 0957.46514)], and is associated with a differential graded algebra generated by the variables and corresponding skew differentials and called a Wess-Zumino differential calculus. By a result of the reviewer, [J. Algebra 174, No. 1, 267-281 (1995; Zbl 0833.16025)], under suitable conditions on the parameters, $A_n(q, P)$ has a simple localization $B_n(q, P)$ which is Noetherian of Krull and global dimension $n$ and may be regarded as a better analogue of the Weyl algebra $A_n$ than $A_n(q, P)$ itself. *L. Rigal*, [Bull. Sci. Math. 121, No. 6, 477-505 (1997; Zbl 0895.17007)], introduced a notion of holonomicity for $B_n(q, P)$-modules and proved an analogue of Bernstein's inequality. In the present paper, given a differential graded algebra morphism between two Wess-Zumino differential calculi, the author constructs, under appropriate conditions on the parameters, inverse and direct image functors between categories of left modules over the corresponding quantum Weyl algebras. The restrictions on the parameters are stronger for direct images than for inverse images because of the need for an involution to switch from right modules to left modules. Holonomicity is defined for modules over $A_n(q, P)$ and results giving sufficient conditions for its preservation by inverse and direct images are given. An analogue of Kashiwara's theorem on the equivalence of categories arising from the direct image for an embedding is given for modules over the simple localized quantum Weyl algebras.

Reviewer: David A.Jordan (Sheffield)

**MSC:**

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|---|---|---|
| 16W35 | Ring-theoretic aspects of quantum groups (MSC2000) | Cited in **2** Documents |
| 16S36 | Ordinary and skew polynomial rings and semigroup rings | |
| 16D30 | Infinite-dimensional simple rings (except as in 16Kxx) | |
| 16D90 | Module categories in associative algebras | |
| 16E45 | Differential graded algebras and applications (associative algebraic aspects) | |
| 16S32 | Rings of differential operators (associative algebraic aspects) | |
| 46L89 | Other "noncommutative" mathematics based on $C^*$-algebra theory | |

**Keywords:**
quantum Weyl algebras; skew differentials; inverse images; holonomicity; differential graded algebras; differential calculi; simple localizations

**Full Text:** DOI

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