Ngwa, G. A.; **Shu, W. S.**

A mathematical model for endemic malaria with variable human and mosquito populations.
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From the paper: We develop and analyse a model that incorporates compartments for mosquito populations. Following the ideas advanced by *J.L. Aron* [Math. Biosci. 64, 249-259 (1983; Zbl 0515.92026); ibid. 88, No. 1, 37-47 (1988; Zbl 0637.92007); ibid. 90, No. 1/2, 385-396 (1988; Zbl 0651.92018)], we introduce in our model a class of persons who are partially immune to the disease malaria, but who may be infectious. We assume density dependent death rates in both vector and human populations so that the total populations are varying with time through a modification of the logistic equation that includes disease related deaths.

A deterministic differential equation model for endemic malaria involving variable human and mosquito populations is analysed. Conditions are derived for the existence of endemic and disease-free equilibria. A threshold parameter $\widetilde{R}_0$ exists and the disease can persist if and only if $\widetilde{R}_0$ exceeds 1. The disease-free equilibrium always exists and is globally stable when $\widetilde{R}_0$ is below 1. Numerical simulations show that the endemic equilibrium, when it exists, is unique and globally stable.

The paper is organised as follows. In Section 2, we briefly outline the derivation of the model and investigate the existence of steady states in Section 3. A linear stability analysis around the steady states is performed in Section 4, where we show that the disease-free equilibrium is globally stable. In Section 5, we present some numerical simulations, and round up the paper with some concluding remarks in Section 6.

MSC:

92D30 Epidemiology
92C60 Medical epidemiology
34C60 Qualitative investigation and simulation of ordinary differential equation models
34D23 Global stability of solutions to ordinary differential equations

Keywords:

endemic equilibrium; simulations; density dependent death rates; endemic malaria; threshold parameter

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