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A smoothness indicator for adaptive algorithms for hyperbolic systems. (English)

Zbl 0998.65092
J. Comput. Phys. 178, No. 2, 323-341 (2002).

Summary: The formation of shock waves in solutions of hyperbolic conservation laws calls for locally adaptive numerical solution algorithms and requires a practical tool for identifying where adaption is needed. In this paper, a new smoothness indicator (SI) is used to identify "rough" solution regions and is implemented in locally adaptive algorithms. The SI is based on the weak local truncation error of the approximate solution. It was recently reported in *S. Karni* and *A. Kurganov* [Local error analysis for approximate solutions of hyperbolic conservation laws, Adv. Comput. Math. 22, No. 1, 79–99 (2005; Zbl 1127.65070)], where error analysis and convergence properties were established.

The present paper is concerned with its implementation in scheme adaption and mesh adaption algorithms. The SI provides a general framework for adaption and is not restricted to a particular discretization scheme. The implementation in this paper uses the central-upwind scheme of *A. Kurganov, S. Noelle*, and *G. Petrova* [SIAM J. Sci. Comput. 23, No. 3, 707–740 (2001; Zbl 0998.65092)].

The extension of the SI to two space dimensions is given. Numerical results in one and two space dimensions demonstrate the robustness of the proposed SI and its potential in reducing computational costs and improving the resolution of the solution.

MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
35L65 Hyperbolic conservation laws
65M50 Mesh generation, refinement, and adaptive methods for the numerical solution of initial value and initial-boundary value problems involving PDEs
76L05 Shock waves and blast waves in fluid mechanics
76M20 Finite difference methods applied to problems in fluid mechanics

Keywords:

hyperbolic conservation laws; local truncation error; smoothness indicator; nonoscillatory central schemes; numerical results; shock waves; central-upwind scheme

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