Tsoularis, A.; **Wallace, J.**

Analysis of logistic growth models. (English) Zbl 0993.92028

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Summary: A variety of growth curves have been developed to model both unpredated, intraspecific population dynamics and more general biological growth. Most predictive models are shown to be based on variations of the classical Verhulst logistic growth equation. We review and compare several such models and analyse properties of interest for these. We also identify and detail several associated limitations and restrictions.

A generalized form of the logistic growth curve is introduced which incorporates these models as special cases. Several properties of the generalized growth are also presented. We furthermore prove that the new growth form incorporates additional growth models which are markedly different from the logistic growth and its variants, at least in their mathematical representation. Finally, we give a brief outline of how the new curve could be used for curve-fitting.

MSC:

92D25 Population dynamics (general)
34C60 Qualitative investigation and simulation of ordinary differential equation models
92B05 General biology and biomathematics

Keywords:

biological growth dynamics; logistic growth; generalized logistic growth; inflection point; incomplete beta-function; beta function; gamma function; mimimax; saddle curve; finite difference method

Software:

GenStat

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