Milton, Graeme W.

The theory of composites. (English) Zbl 0993.74002

Cambridge Monographs on Applied and Computational Mathematics. 6. Cambridge: Cambridge University Press. xxviii, 719 p. (2002).

This book is a mathematically well-based introduction to the theory of composite materials, considering not only mechanical properties but also electrical, thermal, magnetic, thermoelectric, piezoelectric, poroelastic, and electromagnetic properties. It is not only addressed to mathematicians, but also to physicists, geophysicists, material scientists, and electrical and mechanical engineers. Therefore, the book avoids the format of lemmas, propositions, and theorems, but its focus is on explaining the main ideas and providing proofs that avoid excessive mathematical technicalities.

Although a rather wide range of problems of composite materials is treated within the volume of 700 pages, the main focus of this book is on the relations between the underlying microstructure and the resultant effective composite properties. The whole set of treated problems is structured and presented in 31 chapters. The first chapter gives some motivations for studying composites and outlines homogenization theory from various viewpoints. The second chapter introduces the basic equations of interest for composites and presents numerical approaches to solving them. In chapters 3 to 9 results are presented for effective moduli in such microstructural situations where at least some of the effective moduli can be determined exactly. Beyond this, chapter 10 discusses some of the many available approximations for estimating effective moduli. Chapter 11 considers wave propagation for the limit case of very long wavelengths. The general theory of determining effective tensors is covered by chapters 12 to 18. Here, in particular, the following topics are treated: the formulation as a problem in Hilbert space; various variational principles; convergent series expansions for the effective tensor in powers of the variation in local tensor field; how the terms in the series expansion can be expressed in terms of correlation functions; other perturbation solutions; exact relations in composites; and the analytic properties of effective tensor as a function of constituent tensors. The subsequent chapters 19 and 20 are devoted to the Y-tensor and are optional. Chapter 21 introduces the problem of bounding effective tensors. Chapters 22 to 26 describe available variational methods for bounding effective tensors, whereas chapters 27 and 28 show how the analytical properties of the effective tensor lead to large families of bounds. Chapter 29 outlines the parallel between operations on analytic functions and operations in subspace collections, and shows how this leads to bounds for multicomponent composites. Eventually, chapter 30 discusses general properties and effective tensors when the microstructure is varied over all configurations, and chapter 31 shows why problems of bounding effective tensors are equivalent to quasiconvexification problems.

Concluding, it can be stated that this book complements the set of available books on composites and homogenization in a valuable way. The considerations are not restricted to mechanical properties, but are relatively open to any other characteristics. The presentation is of a good mathematical substance without frightening non-mathematicians with excessive intricate mathematical technicalities. Besides, this book also presents numerous new results at the frontier of mathematics and the theory of composites.

Reviewer: Wilfried Becker (Siegen)

MSC:

74-02	Research exposition (monographs, survey articles) pertaining to mechanics of deformable solids
74A40	Random materials and composite materials
74E30	Composite and mixture properties
74Q15	Effective constitutive equations in solid mechanics

Keywords:

composite materials; microstructure; effective moduli; homogenization theory; effective tensors; wave propagation; Hilbert space; series expansions; correlation functions; perturbation solutions; Y-tensor; variational methods; analytic functions; quasiconvexification; bound on effective tensor

Full Text: DOI