Risteski, Ice; **Covachev, Valéry**

Complex vector functional equations. (English) Zbl 0990.39021

Singapore: World Scientific. xi, 324 p. (2001).

Most of the book is about the equation

$$f_1(z_1, z_2, \dots, z_p) + f_2(z_2, \dots, z_p, z_{p+1}) + \dots + f_{n-p+1}(z_{n-p+1}, \dots, z_n) +$$

$$+ f_{n-p+2}(z_{n-p+2}, \dots, z_n, z_1) + f_{n-p+3}(z_{n-p+3}, \dots, z_n, z_1, z_2) + \dots +$$

$$+ f_n(z_n, z_1, \dots z_{n+p-1}) = 0,$$

its particular cases and similar equations (for instance with more than one cycle of variables) and systems of equations, so the title Cyclic Functional Equations seems more appropriate. (On pages 92-94 also the cocycle equation $f(x,y) + f(x+y,z) = f(x,y+z) + f(y,z)$ is briefly discussed.) Moreover, many of the equations and their solutions make sense with variables in an arbitrary set of sufficiently large cardinality and with function values in an abelian group (that would make also one equation out of some systems of equations). Later in the book there is addition and sometimes subtraction in some variables, so there they should be elements of a semigroup or of a group, respectively (occasionally the groups need to be abelian and/or divisible). In some equations variables are multiplied by constants or function values are multiplied by constants or by other function values, so a ring may be the appropriate domain or range. At places where continuity is assumed, some topology is needed. But this reviewer could not find anything that justified the restriction to the complex plane as range of the unknown functions and to its Cartesian powers as domain for the variables. Of the many works (none of them among the references in the present book) dealing with functional equations for which sets of complex numbers or complex manifolds as ranges and/or domains are essential two examples are the papers of *A. Smajdor* and *W. Smajdor* [Math. Z. 98, 235-242 (1967; Zbl 0166.12803)] and of *E. Peschl* and *L. Reich* [Arch. Math. 21, 578-582 (1971; Zbl 0223.32003)].

The proofs are highly technical. The list of references contains papers that appeared before 1968 or after 1998 (the latter by the authors, some in collaboration with K. G. Trenčevski) and books. Much has been published, however, between 1968 and 1998 about cyclic equations [e.g., *O. E. Gheorghiu*, Bull. Math. Soc. Sci. Math. Répub. Soc. Roum., Nouv. Sér. 17(65), 295-300 (1973; Zbl 0308.39007); *D. S. Mitrinović* and *J. Pečarić*, Cyclic inequalities and cyclic functional equations (1991; Zbl 0731.26010)] and about the cocycle equation [e.g., *B. Jessen, J. Karpf*, and *A. Thorup*, Math. Scand. 22, 257-265 (1968; Zbl 0183.04004); *B. R. Ebanks* and *C. T. Ng*, Aequationes Math. 46, No. 1-2, 76-90 (1993; Zbl 0801.39008); etc.].

Remark: On page 122 the authors state "In the more general formulation as given by *D. Ž. Djoković* [Publ. Fac. Electrotech. Univ. Belgrade, Ser. Math. Phys. 143-155, 45-50 (1965; Zbl 0147.12601)] they [Djoković's theorems] are invalid". An e-mail message from I. Risteski to D. Ž. Djoković, dated April 10, 2002, seems to withdraw this claim. Anyway, in this reviewer's opinion such statements should be accompanied by counter examples, or the errors in the proofs should be pointed out.

Reviewer: János Aczél (Waterloo/Ontario)

MSC:

39B32	Functional equations for complex functions
39-02	Research exposition (monographs, survey articles) pertaining to difference and functional equations
30D05	Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable
39B72	Systems of functional equations and inequalities
39B52	Functional equations for functions with more general domains and/or ranges

Keywords:

cyclic functional equations; cocycle equation; Cauchy's equation; complex vector functional equations; textbook; systems