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Entropy splitting and numerical dissipation. (English) Zbl 0987.65086

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This paper addresses some computational aspects of an existing arbitrary order central difference scheme based on flux entropy splitting for systems of hyperbolic conservation laws. In particular, the authors first investigate the choice of the arbitrary parameter which determines the amount of splitting for the problem of a perfect gas. The choice of the parameter is problem dependent. The authors then investigate the influence of the splitting on the nonlinear stability of the central difference scheme for long time integrations of unsteady flows. The paper also investigate the influence of the splitting on the numerical dissipation if such a dissipation is needed to stabilize the central scheme. The techniques for the equations governing a perfect gas are extended to problems with other equation states such as a thermally perfect gas and magnetohydrodynamics. Extensive numerical experiments are performed to demonstrate the effectiveness of the techniques developed.

Reviewer: Song Wang (Nedlands)

MSC:

65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
76N15	Gas dynamics (general theory)
76W05	Magnetohydrodynamics and electrohydrodynamics
76M20	Finite difference methods applied to problems in fluid mechanics
65M15	Error bounds for initial value and initial-boundary value problems involving PDEs
35L65	Hyperbolic conservation laws

Keywords:

entropy variables; systems of hyperbolic conservation laws; thermally perfect gas; flux entropy splitting; central difference scheme; compact schemes; nonlinear stability; shock-capturing methods; nonlinear filters; conservative differencing; unsteady flows; magnetohydrodynamics; numerical experiments

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