Buslaev, Vladimir; Grigis, Alain

Turning points for adiabatically perturbed periodic equations. (English) Zbl 0987.35013

J. Anal. Math. 84, 67-143 (2001).

This work consists of two parts. Part I is devoted to the formal solutions of the general equation

$$i\varepsilon\frac{dy}{dt}=A(t)y,\ t\in\Delta=[\alpha,\beta], \tag{1}$$

where $A(t)$ is a differential operator defined on a space X of a-periodic ($a>0$) functions of x; $\varepsilon>0$, $\varepsilon\to 0$. There are introduced some suitable general assumptions on the structure of the spectrum of $A(t)$; it is defined the turning point as a point with a certain spectral property. Under these assumptions some properties of formal asymptotic solutions are studied.

In Part 2 the authors discuss the special features of the formal solutions of the equation (1) corresponding to the equation

$$-\psi_{xx}+p(x)\psi+v(\varepsilon x)=0, \tag{2}$$

where p and v are smooth real functions; p is also a-periodic. The existence of exact solutions of (2) whose asymptotic behavior is given by the constructed formal solutions, is proved.

Reviewer: Ion Onciulescu (Iaşi)

MSC:

35B10 Periodic solutions to PDEs

Keywords:

formal asymptotic solutions

Full Text: DOI

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