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Inner models and large cardinals. (English) Zbl 0987.03002

de Gruyter Series in Logic and Its Applications. 5. Berlin: de Gruyter. xi, 369 p. (2001).

The book is a well-written and self-contained introduction to the theory of fine structural inner models, an area of set theory concerned with questions about models of set theory. Such models can be seen as a generalization of Gödel's constructible universe L in a way that they reflect certain facts about the structure of the set-theoretic universe. In fact, these models are constructed in a similar way as L is, but relative to coherent extender sequences. Therefore, (canonical) inner models can be constructed when we have large cardinals beyond a measurable without losing too much control of the whole model. To be able to handle such structures, a deep structural theory – the so-called fine structural one – was introduced by Jensen in the early 70's. Using these ideas the whole theory of inner models was established by Dodd, Jensen, Mitchell, Steel and others.

The author gives an introduction to the techniques of this theory; so the book can serve as a textbook even for graduate students. In the first three chapters a detailed introduction to the general fine structural theory of acceptable structures is developed. Acceptable J -structures and their properties are investigated. The author develops only as much theory as is necessary to understand a first application of (rather small) inner models.

In the following five chapters, the author gives the full core model theory of the theory for measures of order zero. This is a nice example of inner models to start with to understand the theory of inner models from the beginning without too many bothering details. At first the reader is provided with the basic elements of the whole theory of inner models, the preface. It can be seen how the (so far linear) iterations work and the main tools for the copying of such iterations are given. After looking at the comparison process, the author is aiming for the main tool, the solidity and condensation lemma. After all, the construction of a canonical extender model, the background certified core model K^c , is developed and finally in Chapter 7 the definition of the core model K is given, its existence is proven and basic properties are discussed.

The advanced reader will be provided with enough details to understand the general theory for models that can contain up to one strong cardinal. This is finally discussed in Chapter 8.

In the remaining last chapter the theory of inner models that are beyond one strong cardinal is described. There the reader can see where the problems are coming into play when one tries to construct an inner model for higher cardinals. That means, the author unravels the complexity of the comparison process which is the main tool in the theory of inner models. This theory is based on the λ -indexing. The author presents Jensen's extender models.

Reviewer: [Martin Weese \(Potsdam\)](#)

MSC:

- 03-02 Research exposition (monographs, survey articles) pertaining to mathematical logic and foundations
- 03E45 Inner models, including constructibility, ordinal definability, and core models
- 03E55 Large cardinals
- 03E35 Consistency and independence results

Cited in **26** Documents

Keywords:

[fine structure](#); [inner models](#); [large cardinals](#); [core model](#); [extender model](#)