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The space-time conservation element and solution element method: A new high-resolution and genuinely multidimensional paradigm for solving conservation laws. (English)

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J. Comput. Phys. 156, No. 1, 89-136 (1999).

The authors derive a new high-resolution and genuinely multidimensional numerical method for solving conservation laws. The method is derived using original integral form of equations, rather than their differential form. The aim is to find such a numerical scheme which is authomatically consistent with properties derived from exact integral or differential forms, e.g. with jump conditions across the shock and with the properties of characteristics.

The method requires that: space and time must be treated as a single entity; both local and global flux conservation in time and space are ensured; no dimension-splitting approach is used. Specifically, the method uses a staggered space-time mesh, avoids the use of Riemann solvers, monotonicity constraints, and special techniques that are not applicable to general problems.

The authors begin with a rigorous exposition of CE/SE schemes, then they evaluate their accuracy on simple scalar linear advection equation and discuss nonreflecting boundary conditions. At the end they present several numerical experiments which illustrate the robustness of the described approach.

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MSC:

76M25 Other numerical methods (fluid mechanics) (MSC2010)
76N15 Gas dynamics (general theory)
76L05 Shock waves and blast waves in fluid mechanics

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space-time conservation element; flux conservation; high-resolution multidimensional method; conservation laws; shocks; contact discontinuities; scalar linear advection equation; nonreflecting boundary conditions

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