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Numerical methods for ordinary differential equations in the 20th century. (English)

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The paper is a survey following many of the main strands in the development of numerical methods for general initial value problems, stiff systems, or special problem types. The early contributions of Bashforth, Adams and Runge, together with an introduction to the fundamental work of Euler form the subject of a first section.

Further sections deal either with specific periods of time or with contributions with a unifying scheme: Heun, Nyström and Moulton papers, Milne's device, Taylor series, modern theory of linear multistep methods and Runge-Kutta methods. Nontraditional methods are also revised.

A special section is dedicated to stiff problems, and another to the beginnings of differential equation software. The last section treats some special problems.

Reviewer: Dana Petcu (Timișoara)

MSC:

- 65L05 Numerical methods for initial value problems involving ordinary differential equations
- 65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
- 01A60 History of mathematics in the 20th century
- 65-03 History of numerical analysis
- 34A34 Nonlinear ordinary differential equations and systems

Cited in **1** Review
Cited in **19** Documents

Keywords:

initial value problems; linear multistep methods; Runge-Kutta methods; stability; convergence; stiff problems; historical survey; Adams-Bashforth method; Euler method; Heun method; Nyström method; Adams-Moulton method; Milne method

Software:

DIFSUB; DASSL

Full Text: [DOI](#)

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