Vasil’ev, A. I.
The inverse problem of best approximation theory in $F$-spaces. (English. Russian original)
Zbl 0965.41019

Let $\{L_n\}_{n \in \mathbb{N}}$ be a sequence of subspaces of a space $X$, $L_n \subset L_{n+1}$. For each $x \in X$ this sequence generates a nonincreasing sequence of numbers $a_n = E(x, L_n)$, convergent to zero where $E(x, L_n)$ is the best approximation of $x$ in $L_n$. The inverse problem of the best approximation theory can be stated as follows: Let $\{a_n\}_{n \in \mathbb{N}}$ be a given nonincreasing convergent to zero sequence of nonnegative numbers and $\{L_n\}_{n \in \mathbb{N}}$ a given sequence of subspaces of a space $X$; there exists an element $x \in X$ such that $a_n = E(x, L_n)$. (Of course, the space $X$ is endowed with a structure which allows to set the problem of the best approximation.)

The author treats this problem in $F$-spaces, i.e. in complete real linear spaces a metric invariant under shift. Additionally, it is assumed that the metric $d$ of the $F$-space $X$ is monotone, i.e. $x \in X \setminus \{\theta\}$ and $0 < \alpha < \beta$ imply $d(\theta, dx) \leq d(\theta, \beta x)$, where $\theta$ is the zero of $X$. The problem in question is examined also in locally pseudo-convex $F$-spaces and in $F$-spaces of mappings of a measure space into an $F$-space.

Reviewer: Borislav Crstici (Timişoara)

MSC:
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
41A50 Best approximation, Chebyshev systems
46B20 Geometry and structure of normed linear spaces

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best approximation