Popov, B. O.

Let $f(x) \in C^{m+1}[a, b]$, $f^{(m+1)}(x) \neq 0$ if $x \in [a, b]$. Then the maximal error $\mu$ of the best Chebyshev approximation by polynomials $P_m(x)$ of order $m$ on the interval $[a, b]$ is equal to

$$\mu = \frac{1}{2^{2m+1}(m+1)!} |f^{(m+1)}(\xi)|(b-a)^{m+1}, \quad \xi \in (a, b).$$

If for some approximating expression $F(A, x)$ we have

$$\mu = \frac{1}{2^{2m+1}(m+1)!} |\eta(f, F)|(b-a)^{m+1},$$

then the function $\eta(f, F)$ is called a kernel of approximation of the function $f(x)$, using the expression $F(A, x)$. The author presents the following result. Let $f(x) \in C^{m+1}[a, b]$, $\eta(f, F) \in \mathbb{C}^1[a, b]$, $w(x) \in C^1[a, b]$ ($w(x)$ is a weight), $\eta(f, F)/w(x) \neq 0$ if $x \in [a, b]$, then for $r \to \infty$ the maximal error $\mu$ of the balance weighted approximation of a function $f(x)$, $x \in [a, b]$ by Chebyshev splines $S(F, x)$ with $r$ links is equal to

$$\mu = \frac{1}{2^{2m+1}r^{m+1}(m+1)!} \left( \int_a^b \frac{\eta(f, F)}{W(x)} \frac{dx}{(m+1)^{m+1}} \right)^{m+1} \left[ 1 + O \left( \frac{b-a}{r} \right) \right].$$

Using this result the author constructs a method of finding of balance approximations as a procedure for the system Maple V. Release 5.

Reviewer: A.D.Borisenko (Kyïv)

MSC:
41A50 Best approximation, Chebyshev systems

Keywords:
computer algebra; balance approximations; best Chebyshev approximation; maximal error

Software:
Maple