del Barrio, Eustasio; **Giné, Evarist**; **Matrán, Carlos**

Central limit theorems for the Wasserstein distance between the empirical and the true distributions. (English) Zbl 0958.60012

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For a sequence of i.i.d. random variables, it is well-known that the Wassertein distance between their empirical distribution and the true distribution tends to zero. The rate of this convergence is studied. As a by-product, some limit theorems for the Ornstein-Uhlenbeck processes are also derived.

Reviewer: D.Tu (Kingston)

MSC:

60F05 Central limit and other weak theorems
60F17 Functional limit theorems; invariance principles
62E20 Asymptotic distribution theory in statistics
62G30 Order statistics; empirical distribution functions

Keywords:

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References:

[1] Araujo, A. and Giné, E. (1980). The Central Limit Theorem for Real and Banach Valued Random Variables. Wiley, New York. · Zbl 0457.60001

[2] Billingsley, P. (1968). Convergence of Probability Measures. Wiley, New York. · Zbl 0172.21201

[3] Borell, C. (1975). The Br ünn-Minkowski inequality in Gauss space. Invent. Math. 30 207-216. · Zbl 0311.60007 · doi:10.1007/BF01425510

[4] Byczkowski, T. (1977). Gaussian measures on Lp spaces 0 p < . Studia Math. 59 249-261. · Zbl 0379.60007

[5] Cs örg o, M. and Horváth, L. (1986). Approximations of weighted empirical and quantile processes. Statist. Probab. Lett. 4 275-280. Cs örg o, M. and Horváth, L. (1988a). On the distributions of Lp norms of weighted uniform empirical and quantile processes. Ann. Probab. 16 142-161. Cs örg o, M. and Horváth, L. (1988b). Central limit theorems for Lp-norms of density estimators. Probab. Theory Related Fields 80 269-291. · Zbl 0676.60042 · doi:10.1016/0167-7152(86)90043-X

[6] Cs örg o, M. and Horváth, L. (1993). Weighted Approximations in Probability and Statistics. Wiley, New York. Cs örg o, M., Cs örg o, S., Horváth, L. and Mason, D. M. (1986a). Weighted empirical and quantile processes. Ann. Probab. 14 31-85. Cs örg o, M., Cs örg o, S., Horváth, L. and Mason, D. M. (1986b). Normal and stable convergence of integral functions of the empirical distribution function. Ann. Probab. 14 86-118. · Zbl 0589.60029

[7] Cs örg o, M., Horváth, L. and Shao, Q. M. (1993). Convergence of integrals of uniform empirical and quantile processes. Stochastic Process. Appl. 45 283-294. · Zbl 0784.60038 · doi:10.1016/0304-4149(93)90075-F

[8] Cs örg o, S. and Mason, D. (1991). A probabilistic approach to the tails of infinitely divisible laws. In Sums, Trimmed Sums and Extremes 317-336. Birkhäuser, Boston. · Zbl 0728.60019

[9] de Acosta, A., Araujo, A. and Giné, E. (1978). Poisson measures, Gaussian measures and the central limit theorem in Banach spaces. In Advances in Probability (J. Kuelbs, ed.) 4 1-68. Dekker, New York.

[10] de Acosta, A. and Giné, E. (1979). Convergence of moments and related functionals in the central limit theorem in Banach spaces.Wahrsch. Verw. Gebiete 48 213-231. · Zbl 0388.60008 · doi:10.1007/BF01886874

[11] Feller, W. (1971). An Introduction to Probability Theory and Its Applications 2. Wiley, New York. · Zbl 0219.60003

[12] Giné, E. (1983). The Lévy-Lindeberg central limit theorem in Lp, 0 < p < 1. Proc. Amer. Math. Soc. 88 147-153.

[13] Giné, E. and Zinn, J. (1983). Central limit theorems and weak laws of large numbers in certain Banach spaces.Wahrsch. Verw. Gebiete 62 323-354. · Zbl 0488.60009 · doi:10.1007/BF00535258

[14] Horn, R. A. (1972). On necessary and sufficient conditions for an infinitely divisible distribution to be normal or degenerate.Wahrsch. Verw. Gebiete 21 179-187. · Zbl 0213.20402 · doi:10.1007/BF00538390

[15] Jain, N. C. (1977). Central limit theorems and related questions in Banach space. In Proceedings of Symposium in Pure and Applied Mathematics 31 55-65. Amer. Math. Soc. Providence, RI. · Zbl 0389.60002

[16] Lawniczak, A. (1983). The Lévy-Lindeberg central limit theorem in Orlicz spaces. Proc. Amer. Math. Soc. 89 673-679. · Zbl 0542.60027 · doi:10.2307/2044604

[17] Ledoux, M. and Talagrand, M. (1991). Probability in Banach Spaces. Springer, Berlin. · Zbl 0748.60004

[18] Mandl, P. (1968). Analytical Treatment of One-Dimensional Markov Processes. Springer, New York. · Zbl 0179.47802

[19] Mandrekar, V. and Zinn, J. (1980). Central limit problem for symmetric case: convergence to non-Gaussian laws. Studia Math. 67 279-296. · Zbl 0461.60022

[20] Mason, D. M. (1991). A note on weighted approximations to the uniform empirical and quantile processes. In Sums, Trimmed Sums and Extremes 269-283. Birkhäuser, Boston. · Zbl 0722.60042

[21] Mason, D. M. (1998). An exponential inequality for a weighted approximation to the uniform empirical process with applications.

[22] Montgomery-Smith, S. J. (1994). Comparison of sums of independent identically distributed random variables. Probab. Math. Statist. 14 281-285. · Zbl 0827.60005

[23] Pisier, G. (1983). Some applications of the metric entropy condition to harmonic analysis. Banach Spaces, Harmonic Analysis and Probability. Lecture Notes in Math. 995 123-154. Springer, Berlin. · Zbl 0517.60043

[24] Pisier, G. (1986). Probabilistic methods in the geometry of Banach spaces. Probability and Analysis. Lecture Notes in Math. 1026 167-241. Springer, Berlin. · Zbl 0606.60008

[25] Resnick, S. I. (1987). Extreme Values, Regular Variation and Point Processes. Springer, New York. · Zbl 0633.60001

[26] Shorack, G. R. and Wellner, J. A. (1986). Empirical Processes with Applications to Statistics. Wiley, New York. · Zbl 1170.62365

[27] Sudakov, V. N. and Tsirel'son, B. S. (1974). Extremal properties of half-spaces for spherically invariant measures. Zap. Nau cn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) 41 14-24.

[28] Talagrand, M. (1996). New concentration inequalities on product spaces. Invent. Math. 126 505- 563. · Zbl 0893.60001 · doi:10.1007/s002220050108

[29] Uchiyama, K. (1980). Brownian first exit from and sojourn over one-sided moving boundary and application.Wahrsch. Verw. Gebiete 54 75-116. · Zbl 0431.60080 · doi:10.1007/BF00535355