Churchill, R. C.

Two generator subgroups of SL$(2, \mathbf{C})$ and the hypergeometric, Riemann, and Lamé equations. (English) Zbl 0958.34074

J. Symb. Comput. 28, No. 4-5, 521-545 (1999).

From the introduction: A subgroup $G \subset \mathrm{SL}(2, \mathbb{C})$ has precisely one of the following four properties: (a) the projective representation fixes a line; (b) the projective representation permutes two lines, fixing neither; (c) the projective group is isomorphic to the alternating group A_4, the symmetric group S_4, or the alternating group A_5; or (d) the Zariski closure of G is $\mathrm{SL}(2, \mathbb{C})$.

Summary: For the purpose of constructing explicit solutions to second-order linear homogeneous differential equations on the Riemann sphere the Kovacic algorithm partitions the subgroups of $\mathrm{SL}(2, \mathbb{C})$ into four classes and initially determines which class contains the differential Galois group of the input equation. The author proves in the case of the hypergeometric and Riemann equations that the relevant class can be determined directly from the coefficients by elementary calculation. He also treats the (nonalgebraic form of the) Lamé equation, to which the Kovacic algorithm is not directly applicable. In that instance he combines the Kovacic results with his to produce an algorithm for determining the class of the associated group.

From the group-theoretic viewpoint the problem solved herein is the following: given arbitrary $S, T \in \mathrm{SL}(2, \mathbb{C})$, determine which class contains the group $\langle S, T\rangle$ generated by S and T.

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MSC:

34M15	Algebraic aspects (differential-algebraic, hypertranscendence, group-theoretical) of ordinary differential equations in the complex domain
34A26	Geometric methods in ordinary differential equations
14D21	Applications of vector bundles and moduli spaces in mathematical physics (twistor theory, instantons, quantum field theory)
34M45	Ordinary differential equations on complex manifolds
20G42	Quantum groups (quantized function algebras) and their representations
22E70	Applications of Lie groups to the sciences; explicit representations

Keywords:

explicit solutions; second-order linear homogeneous differential equations; differential Galois group; hypergeometric and Riemann equations; Lamé equation; Kovacic algorithm

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