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The \mathcal{A} -hypergeometric system associated with a monomial curve. (English) Zbl 0952.33009
Duke Math. J. 99, No. 2, 179-207 (1999).

Let \mathcal{A} be a spanning subset of \mathbb{Z}^{n+1} consisting of r elements, and let $\alpha \in \mathbb{C}^{n+1}$. In the late eighties Gel'fand, Kapranov and Zelevinskij associated with \mathcal{A} and α a holonomic system of differential equations in \mathbb{C}^r , called the \mathcal{A} -hypergeometric system with exponent (or parameter) α . Its solutions are called the \mathcal{A} -hypergeometric functions with parameter α [see *I. M. Gel'fand, A. V. Zelevinskij* and *M. M. Kapranov*, *Funct. Anal. Appl.* 23, No. 2, 94-106 (1989; [Zbl 0721.33006](#)); *Adv. Math.* 84, No. 2, 255-271 (1990; [Zbl 0741.33011](#))]. In the literature \mathcal{A} -hypergeometric systems are also called GKZ-systems. The paper under review studies the case of \mathcal{A} -hypergeometric systems associated with monomial curves, which corresponds to the case $n = 1$. All rational \mathcal{A} -hypergeometric functions with parameter α are shown to be Laurent polynomials. This property is proven by counterexample not to be true in the general case $n > 1$. The rational \mathcal{A} -hypergeometric functions with parameter $\alpha \in \mathbb{Z}^2$ are shown to span a space of dimension at most 2. The value 2 is attained if and only if the monomial curve is not arithmetically Cohen-Macaulay. For all values of α , the holonomic rank $r(\alpha)$ of the system is proven to satisfy the inequalities $d \leq r(\alpha) \leq d + 1$. Moreover $r(\alpha) = d + 1$ exactly for all $\alpha \in \mathbb{Z}^2$ for which the space of rational solutions has dimension 2. The inequalities for the holonomic rank have also been obtained using different methods by *M. Saito, B. Sturmfels* and *N. Takayama* [Gröbner deformations of hypergeometric differential equations. Springer-Verlag (2000; [Zbl 0946.13021](#))].

Reviewer: [A.Pasquale \(Clausthal-Zellerfeld\)](#)

MSC:

[33C70](#) Other hypergeometric functions and integrals in several variables
[14D99](#) Families, fibrations in algebraic geometry
[32G99](#) Deformations of analytic structures
[33D15](#) Basic hypergeometric functions in one variable, ${}_r\phi_s$

Cited in **16** Documents

Keywords:

[GKZ-systems](#); [\$\mathcal{A}\$ -hypergeometric function](#); [\$\mathcal{A}\$ -hypergeometric system](#)

Full Text: [DOI](#)

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