**Brokate, Martin**; **Sprekels, Jürgen**
**Hysteresis and phase transitions.** (English) Zbl 0951.74002
Applied Mathematical Sciences. 121. New York, NY: Springer. x, 357 p. (1996).

As the title says, the book is mainly devoted to hysteresis phenomena in phase transition processes. However, such analysis is preceded by an extensive illustration of the mathematical theory of hysteresis in a general context, with plenty of physical examples.

Very appropriately, the first chapter collects the most relevant mathematical tools from measure theory, functional analysis, etc., to be used in the sequel. Then the authors go through a sequence of examples that reach two goals at the same time: to introduce the reader into the subject in a gradual and smooth way, and to trace some history of the problem, referring in particular to the basic works of Madelung, Prandtl and Preisach. This material is taken as a basis to move to the definition of the general hysteresis operator, that in such a way looks well motivated, showing how it incorporates many examples. Then the analysis is focussed on the specific cases of the so-called play operator and of the elastic-plastic element, as tools for constructing more complicated operators of practical interest. All this material is used to study operators of Preisach type and, as a special case, of Prandtl type.

It is effectively remarked that in many cases which are relevant to applications, the basic role is played by the action of rate-independent functionals on vectors of an arbitrarily large number of components (strings), and this fact is largely and advantageously exploited in the exposition. Before passing to the phase change, the authors study the hysteresis in ODEs (for instance, the oscillations of elastic-plastic string), and in two classes of PDEs: the heat equation and the wave equation. For heat equation, they consider the initial-boundary value problem with hysteresis $\frac{\partial y}{\partial t}+\frac{\partial w}{\partial t}-\Delta y=f$ in $\Omega\times(0,t_E)$, where $w$ is the result of the action of an hysteresis operator on $y$. In the hyperbolic case, the equation studied is of the type $\rho\frac{\partial^2 u}{\partial x^2}=\frac{\partial\sigma}{\partial x}$, where $\sigma$ is obtained by applying an hysteresis operator to $\frac{\partial u}{\partial x}$.

The main subject of the book, hysteresis in phase change, is approached with a preparatory chapter 4, which is a concise, but exceptionally clear, review on phase transition, dealing with topics of increasing difficulty. First come the space-independent models in which the free energy density is taken in the form $F=\sum_{i=0}^{\infty}F_i(T)e^i$, where $e$ is a (scalar) order parameter, and special cases of Landau and Devonshire forms are discussed, showing the possible occurrence of hysteresis. Next the phase field models are deduced from the Landau-Ginzburg functional, arriving at the Cahn-Allen equation (for dynamics in which the physical quantity associated with $e$ is not conserved), and to the Cahn-Hilliard equation (in the opposite case), for general forms of the function $F(e,T)$. Specifying the latter, the model of Penrose-Fife and the model of Caginalp are obtained.

The final part of the book is devoted to the analysis of the most interesting models exhibiting hysteresis. So, chapter 5 deals with shape memory alloys (the typical case of austenite-martensite transitions described by the Falk model); the following two chapters analyse the Caginalp model and the Penrose-Fife model, supplemented with energy balance. Finally, the last chapter illustrates a multi-component case (phase transition in eutectoid carbon steel), in which the internal energy is a combination of the volume fractions $a,p,m$ of austenite, pearlite and martensite with temperature-dependent coefficients. To the corresponding energy balance equation the authors add suitable phase transition laws, relating the parameters $a,p,m$ (whose sum is one) to time and temperature (first in the isothermal case, and next for general evolution processes).

The aim of the set of the conclusive chapters on hysteresis in phase transitions is not only to present and motivate the models, but also to show their well-posedness, to illustrate suitable numerical methods, and to comment on the results from the physical point of view, thus providing a comprehensive view of the subject. In conclusion, this is a book of exceptionally high quality for a number of reasons: the selection of the subjects, the way the material is organized, and the style of exposition.

Absolutely recommendable to anybody interested in phase transitions and related topics.

Reviewer: Antonio Fasano (Firenze)

**MSC:**

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| 74-02 | Research exposition (monographs, survey articles) pertaining to mechanics of deformable solids |
| 74N30 | Problems involving hysteresis in solids |
| 35K60 | Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations |
| 35L50 | Initial-boundary value problems for first-order hyperbolic systems |
| 35Q82 | PDEs in connection with statistical mechanics |
| 80A22 | Stefan problems, phase changes, etc. |



**Keywords:**

nonlinear parabolic equations; Preisach-type operators; Prandtl-type operators; multi-component phase transition; play operator; elastic-plastic element; rate-independent functionals; heat equation; wave equation; initial-boundary value problem; hysteresis; phase transition; free energy density; Landau-Ginzburg functional; Cahn-Allen equation; Cahn-Hilliard equation; austenite-martensite transitions; Falk model; Caginalp model; Penrose-Fife model