Kremer, Philip

Propositional quantification in the topological semantics for S4. (English) Zbl 0949.03020

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Summary: K. Fine and S. Kripke extended S5, S4, S4.2 and such to produce propositionally quantified systems such as $\mathrm{S5}\pi+, \mathrm{S4}\pi+$, and $\mathrm{S4.2}\pi+$: Given a Kripke frame, the quantifiers range over all the sets of possible worlds. $\mathrm{S5}\pi+$ is decidable and, as Fine and Kripke showed, many of the other systems are recursively isomorphic to second-order logic. In the present paper we consider the propositionally quantified system that arises from the topological semantics for S4, rather than from the Kripke semantics. The topological system, dubbed $\mathrm{S4}\pi t$, is strictly weaker than its Kripkean counterpart. We prove that second-order arithmetic can be recursively embedded in $\mathrm{S4}\pi t$. In the course of the investigation, we also sketch a proof of Fine's and Kripke's result that the Kripkean system $\mathrm{S4}\pi+$ is recursively isomorphic to second-order logic.

MSC:

03B45 Modal logic (including the logic of norms)

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Keywords:

propositional quantification; modal logic S4; topological semantics; second-order arithmetic; second-order logic

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