Zhang, Shaoyi
Existence of the optimal measurable coupling and ergodicity for Markov processes. (English)
Zbl 0938.60011

This paper studies Markovian coupling for a given transition function $P(x, dy)$ on a Polish space $(E, \rho, \mathcal{E})$, where ρ is a metric on E. The author proves that for given two transition probabilities $P_1(x_1, dy_1)$ and $P_2(x_2, dy_2)$, there always exists a coupled transition probability $P(x_1, x_2, dy_1, dy_2)$ such that

$$\int \rho(y_1, y_2) P(x_1, x_2, dy_1, dy_2) = W(P_1(x_1, \cdot), P_2(x_2, \cdot))$$

for all $x_1, x_2 \in E$, where $W(P_1, P_2)$ is the Wasserstein distance of probability measures P_1 and P_2. Originally, the problem comes from the well-known Dobrushin-Shlosman uniqueness theorem for random fields. In the original proof, the measurability of $P(x_1, x_2, dy_1, dy_2)$ in (x_1, x_2) was missed. See also the reviewer's book "From Markov chains to non-equilibrium particle systems" (1992; Zbl 0753.60055), Theorem 10.9 and §10.8. Very recently, in a forthcoming paper [Acta Math. Sin., Engl. Ed.], the author extends the above result to nonnegative, lower semi-continuous function instead of distance ρ. This enables the author to solve an open problem about stochastic comparison problem. Refer to the reviewer's paper [Acta Math. Sin., New Ser. 10, No. 3, 260-275 (1994; Zbl 0813.60068)] for further background of the study on optimal couplings.

Reviewer: Chen Mu-fa (Beijing)

MSC:

60B05	Probability measures on topological spaces
60K35	Interacting random processes; statistical mechanics type models; percolation theory

Cited in **2** Documents

Keywords:

coupling; measurability; Dobrushin-Shlosman uniqueness theorem

Full Text: DOI

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