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Rigidity and topological conjugates of topologically tame Kleinian groups. (English)

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A finitely-generated torsion-free Kleinian group Γ is topologically tame if H^3/Γ is homeomorphic to the interior of a compact 3-manifold. The main results here are quasiconformal rigidity theorems for such groups. More precisely the author proves that, if a homeomorphism $h : H^3/\Gamma_1 \rightarrow H^3/\Gamma_2$ preserves cusps, geometrically finite ends and ending laminations of geometrically infinite ends and an additional condition (IR), then Γ_1, Γ_2 are $q - c$ conjugate. The condition (IR) is that the injectivity radius at all points of both manifolds is bounded below. If Γ is freely indecomposable as a group, combining results of Bonahon and Thurston, shows that Γ is topologically tame. For freely indecomposable groups Γ_1, Γ_2 also satisfying (IR) the rigidity theorem above had been proved by *Y. N. Minsky* [J. Am. Math. Soc. 7, No. 3, 539-588 (1994; Zbl 0808.30027)]. Thus the extension in this paper is to topologically tame groups which may be freely decomposable. Essential use is made of a branched-covering technique of Canary to extend Bonahon's theory to freely decomposable groups. A particular application of this shows that, if two topologically tame Kleinian groups, which are not free and satisfy (IR) are topologically conjugate then they are $q - c$ conjugate.

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