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**Integral representations over isotropic submanifolds and equations of zero curvature.** (English) [Zbl 0926.53030](#)

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The following setting is considered. There is a submanifold  $\Lambda$  of the cotangent bundle  $T^*M$  of a Riemannian manifold  $M$  which is invariant under the Hamiltonian flow and which is isotropic, i.e., the form  $pdq$  is closed on  $\Lambda$ . The authors work with a global integral representation for quasimodes for the quantum Hamiltonian. This leads to certain geometrical objects over an isotropic submanifold.

Reviewer: [H.-B.Rademacher \(Leipzig\)](#)

**MSC:**

[53D05](#) Symplectic manifolds (general theory)

[53C40](#) Global submanifolds

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[isotropic submanifold](#); [integral representation](#); [quasimodes](#); [geometric quantization](#); [symplectic connection](#)

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