Raymond, J. P.; **Zidani, H.**

Hamiltonian Pontryagin's principles for control problems governed by semilinear parabolic equations. (English) Zbl 0922.49013

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The system is

$$\frac{\partial y}{\partial t} + Ay + f(t,x,y,u) = 0,$$

where A is a second order elliptic operator in an m-dimensional domain Ω with boundary Γ and $0 \le t \le T$. Besides the distributed control u, there are controls v, w acting on the initial and the boundary condition,

$$y(0) = w \quad (x \in \Omega), \qquad \frac{\partial y}{\partial n_A} + g(t,x,y,v) = 0 \quad (x \in \Gamma).$$

Here u (resp. v, w) belongs to $L^q((0,T)\times\Omega)$ (resp. $L^r((0,T)\times\Gamma)$, $L^\infty(\Omega)$). The controls are subject to the pointwise constraints

$$u(t,x) \in K_U, \quad v(t,x) \in K_V, \quad w(x) \in K_W.$$

The cost functional undergoing minimization is

$$J(y,u,v,w) = \int_{(0,T)\times\Omega} F(t,x,y,u)dxdt + \int_{(0,T)\times\Gamma} G(t,x,y,v)d\sigma dt + \int_\Omega \ell(x,y(T),w)dx.$$

The authors define three Hamiltonians (resp. *distributed*, *boundary* and *initial*) by

$$\begin{aligned} H_d(t,x,y,u,p) &= F(t,x,y,u) - pf(t,x,y,u), \\ H_b(t,x,y,v,p) &= G(t,x,y,v) - pg(t,x,y,v), \\ H_i(x,y,w,p) &= \ell(x,y,w) + pw, \end{aligned}$$

and obtain necessary conditions for optimal controls $\bar u(t,x)$, $\bar v(t,x)$, $\bar w(x)$ and optimal solutions $\bar y(t,x)$ in the form of three Pontryagin's principles. These minimum principles are *decoupled* in the sense that each one involves only one of the controls. Since both the distributed and the boundary control may be unbounded, the proof is technically complicated and needs new regularity results for linear and nonlinear parabolic equations.

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MSC:

49K20	Optimality conditions for problems involving partial differential equations	Cited in **53** Documents
93C20	Control/observation systems governed by partial differential equations	
35K15	Initial value problems for second-order parabolic equations	
35K20	Initial-boundary value problems for second-order parabolic equations	

Keywords:

Pontryagin's minimum principle; semilinear parabolic equations; distributed control; boundary control; unbounded controls

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