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**The quantum cohomology ring of flag varieties.** (English) Zbl 0920.14027  
Trans. Am. Math. Soc. 351, No. 7, 2695-2729 (1999).

Summary: We describe the small quantum cohomology ring of complete flag varieties by algebro-geometric methods, as presented in our previous work [*I. Ciocan-Fontanine*, Int. Math. Res. Not. 1995, No. 6, 263-277 (1995; [Zbl 0847.14011](#))]. We also give a geometric proof of the quantum Monk formula.

**MSC:**

[14M15](#) Grassmannians, Schubert varieties, flag manifolds  
[14N10](#) Enumerative problems (combinatorial problems) in algebraic geometry  
[14F99](#) (Co)homology theory in algebraic geometry  
[81T10](#) Model quantum field theories

Cited in **10** Documents

**Keywords:**

quantum cohomology; flag varieties; hyperquot schemes; degeneracy loci; quantum Monk formula

**Full Text:** [DOI](#)

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