Soma, Teruhiko

Existence of non-Banach bounded cohomology. (English) Zbl 0893.55004

Topology 37, No. 1, 179-193 (1998).

For any discrete group G and $n \in \mathbb{Z}$ the bounded cohomology group $H_b^n(G;\mathbb{R})$ admits a canonically defined pseudonorm $\|\cdot\|$. The paper gives a negative answer to the question whether $\|\cdot\|$ is a norm, i.e. $(H_b^n(G;\mathbb{R}), \|\cdot\|)$ is a Banach space. The author shows that $H_b^3(\mathbb{Z} * \mathbb{Z}, \mathbb{R}, \|\cdot\|)$ is not a Banach space (Theorem 1). Moreover, for any discrete group G admitting a surjective homomorphism $f : G \to \mathbb{Z} * \mathbb{Z}$, $(H_b^3(G, \mathbb{R}), \|\cdot\|)$ is not a Banach space (Corollary). The author establishes that for $n \geq 5$ there exists a finitely generated discrete group G such that $(H_b^n(G;\mathbb{R}), \|\cdot\|)$ is not a Banach space (Theorem 2). The proofs are very dense and use some interesting arguments of hyperbolic geometry as well as a result of S. Matsumoto and S. Morita.

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MSC:

55N35	Other homology theories in algebraic topology	Cited in **3** Documents
57M07	Topological methods in group theory	
20J05	Homological methods in group theory	

Keywords:

hyperbolic structure of the figure-eight-knot complement in S^3; K-uniform boundary condition; bounded cohomology

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