Rockafellar, R. Tyrrell; Wets, Roger J.-B.
Variational analysis. (English) Zbl 0888.49001
Grundlehren der Mathematischen Wissenschaften. 317. Berlin: Springer. xiii, 733 p. (1998).

This book lies in the mainstream of present-day research in optimization, equilibrium, and stability of nonlinear systems.

The first two chapters provide an efficient introduction to convex analysis, and set up the basic conventions regarding the arithmetics of extended-real-valued functions. Chapter 3 proposes a new presentation of what has been traditionally called "asymptotic" analysis of sets and functions. The authors explain in detail why the term "horizon" should be preferred to "asymptotic". Chapter 4 discusses the theory of set convergence. The traditional way of defining the convergence of a sequence of sets $\{C^k : k \in \mathbb{N}\}$, is by imposing the coincidence between the inner limit $\liminf C^k$ and the outer limit $\limsup C^k$. This mode of convergence has some serious limitations, specially if nonconvex unbounded sets are involved. The authors remediate this situation by introducing also the horizon inner limit and the horizon outer limit. Chapter 5 discusses the convergence and continuity properties of set-valued mappings. Chapter 6 provides an elegant introduction to tangent and normal cones in a nonconvex nonsmooth context. Epigraphical limits of extended-real-valued functions are discussed in Chapter 7. The tools developped in Chapters 6 and 7 are then combined to handle several important issues that arise in nonsmooth optimization theory: from subderivatives and subgradients of nonsmooth functions to graphical derivatives and coderivatives of set-valued mappings (cf. Chapter 8). Chapter 9 discusses another topic which has received a great deal of attention in the last decade: Lipschitzian properties and metric regularity of set-valued mappings. Other topics presented in this book include: subdifferential calculus (Chapter 10), convex and nonconvex duality theory (Chapter 11), and monotone mappings (Chapter 12). A theory of second-order subdifferentiation based on epigraphical limits is developed in Chapter 13. The books finishes with a review on integral functionals and measurability issues.

Progress in mathematical research is often achieved by unifying seemingly different topics. In my opinion, the authors have succeded in providing a systematic and unified exposition of various issues that lie within the realm of variational analysis. Horizon cones and horizon limits are perhaps the big winners in this presentation. These concepts appear in force each time there is a lack of compactness in the data.

It would not be fair to say that this book is just a good and readable introduction to the subject. In fact, this book also makes many substantial contributions to the further development of the theory itself.

Reviewer: A.Seeger (Avignon)

MSC:

49-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to calculus of variations and optimal control
49J52	Nonsmooth analysis
47H04	Set-valued operators
49J40	Variational inequalities
49J45	Methods involving semicontinuity and convergence; relaxation
49K40	Sensitivity, stability, well-posedness
49N15	Duality theory (optimization)
52A41	Convex functions and convex programs in convex geometry
54C60	Set-valued maps in general topology
90C31	Sensitivity, stability, parametric optimization

Keywords:

convex analysis; nonsmooth optimization; subdifferential calculus

Full Text: DOI