Zhang, Shaoyi; **Xu, Kan**

On the existence of the optimal measurable coupling of transition probability. (Chinese. English summary) Zbl 0884.60088

Acta Math. Sin. 40, No. 1, 5-13 (1997).

This paper studies Markovian coupling for a given transition function $P(x, dy)$ on a Polish space $(E, \rho, \mathcal{E})$, where ρ is a metric on E. Roughly speaking, the author proves that if the family $\mathcal{M} := \{P(x, \cdot) : x \in E\}$ is tight, then there exists a coupled transition probability $P(x_1, x_2, dy_1, dy_2)$ such that

$$\int \rho(y_1, y_2) P(x_1, x_2, dy_1, dy_2) = W(P(x_1, \cdot), P(x_2, \cdot))$$

for all $x_1, x_2 \in E$, where $W(P_1, P_2)$ is the Wasserstein distance of P_1 and P_2. Originally, the problem comes from the well-known Dobrushin-Shlosman uniqueness theorem for random fields. In the original proof, the measurability of $P(x_1, x_2, dy_1, dy_2)$ in (x_1, x_2) was missed. See also the reviewer's book "From Markov chains to non-equilibrium particle systems" (1992; Zbl 0753.60055), Theorem 10.9 and §10.8. Very recently, in a forthcoming paper, the authors improved the above result by removing the tightness assumption. Thus, the authors finally established the existence theorem of ρ-optimal Markovian coupling for time-discrete Markov processes. Refer to the reviewer's paper [Acta Math. Sin., New Ser. 10, No. 3, 260-275 (1994; Zbl 0813.60068)] for further background of the study on optimal couplings. The authors have also extended the above result to the time-continuous jump processes.

Reviewer: Chen Mu-fa (Beijing)

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory

60H05 Stochastic integrals

Keywords:

coupling; measurability; Dobrushin-Shlosman uniqueness theorem