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Hardy spaces of solenoidal vector fields, with applications to the Navier-Stokes equations.

(English) Zbl 0883.35088
Kyushu J. Math. 50, No. 1, 1-64 (1996).

In this very detailed paper the author investigates decay properties of weak solutions both for stationary and nonstationary Navier-Stokes equations. The first part of the paper is concerned with the Cauchy problem to the nonstationary Navier-Stokes equations

$$\frac{\partial u}{\partial t} + u \cdot \nabla u - \Delta u + \nabla p = 0, \quad \nabla \cdot u = 0, \quad x \in \mathbb{R}^n,\ t > 0, \quad u|_{t=0} = a. \tag{1}$$

Using some specific properties of Hardy spaces the author proves that for a suitable initial condition a the weak solution to the problem (1) satisfies the relations $\lim |u(t)|_{L^1(\mathbb{R}^n)} = 0$, $\lim |u(t)|_{H^1(\mathbb{R}^n)} = 0$ as $t \to \infty$. The stationary problem

$$u \cdot \nabla u - \Delta u + \nabla p = \nabla F, \quad \nabla \cdot u = 0, \quad \text{in } \mathbb{R}^n, \quad \lim u = 0 \text{ as } |x| \to \infty \tag{2}$$

is considered in the second part of the paper. It is shown that if $n \geqslant 3$ and F is small in $L^{n/2}$ then the problem (2) has a weak solution u such that

$$u \in L^n(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n), \quad \nabla u \in L^{n/2}(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n).$$

More explicit results are obtained for $n \geq 4$.

Reviewer: I.Sh.Mogilevskij (Ferrara)

MSC:

35Q30 Navier-Stokes equations
35B40 Asymptotic behavior of solutions to PDEs
42B20 Singular and oscillatory integrals (Calderón-Zygmund, etc.)

Keywords:

Hardy spaces; weak solution; decay

Full Text: DOI