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An estimate of a Dirichlet series of Kloosterman type. (English) Zbl 0880.11043

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Let Γ be a finitely generated Fuchsian group of the first kind with the cusp ∞ , and let $q > 0$ be minimal such that $\begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix}$ generates Γ_∞ . For $m \in \mathbb{Z}$, $m \neq 0$ the author defines the exponential sum

$$S(m, c, \Gamma) := \sum_{0 \leq a < qc} \exp\left(2\pi i \frac{ma}{qc}\right), \quad \begin{pmatrix} a & * \\ c & d \end{pmatrix} \in \Gamma \quad (c > 0)$$

and the Dirichlet series

$$\Phi_m(s, \Gamma) := \sum_{c > 0} S(m, c, \Gamma) c^{-2s}.$$

This series comes up in the m -th Fourier coefficient of the Fourier expansion of the Eisenstein series for Γ at the cusp ∞ , and this series also comes up in the zeroth Fourier coefficient of the nonholomorphic Poincaré series $P_m(z, s, \Gamma)$.

The aim of the paper under review is to show that Φ_m admits a meromorphic continuation to the region $\text{Res} > \frac{1}{2}$ and to prove a growth estimate for this function in the domain $\frac{1}{2} < \text{Res} < M$, $|\text{Im}s| \geq 1$. To this end the author computes the inner product of $P_m(z, s, \Gamma)$ with a certain series $E_b(z, s, \Gamma)$ which is closely related with the Eisenstein series. Since Φ_m comes up in the constant term of P_m , this inner product can be expressed in terms of Φ_m .

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MSC:

11F37 Forms of half-integer weight; nonholomorphic modular forms

11F66 Langlands L -functions; one variable Dirichlet series and functional equations

11F30 Fourier coefficients of automorphic forms

Keywords:

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