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An estimate of a Dirichlet series of Kloosterman type. (English) ┃Zbl 0880.11043┃

Let Γ be a finitely generated Fuchsian group of the first kind with the cusp ∞, and let $q > 0$ be minimal such that $\left(\begin{smallmatrix} 1 & q \\ 0 & 1 \end{smallmatrix}\right)$ generates Γ_∞. For $m \in \mathbb{Z}$, $m \neq 0$ the author defines the exponential sum

$$S(m, c, \Gamma) := \sum_{0 \leq a < qc} \exp\left(2\pi i \frac{ma}{qc}\right), \quad \begin{pmatrix} a & * \\ c & d \end{pmatrix} \in \Gamma \quad (c > 0)$$

and the Dirichlet series

$$\Phi_m(s, \Gamma) := \sum_{c>0} S(m, c, \Gamma) c^{-2s}.$$

This series comes up in the m-th Fourier coefficient of the Fourier expansion of the Eisenstein series for Γ at the cusp ∞, and this series also comes up in the zeroth Fourier coefficient of the nonholomorphic Poincaré series $P_m(z, s, \Gamma)$.

The aim of the paper under review is to show that Φ_m admits a meromorphic continuation to the region $\mathrm{Re}\,s > \frac{1}{2}$ and to prove a growth estimate for this function in the domain $\frac{1}{2} < \mathrm{Re}\,s < M$, $|\mathrm{Im}\,s| \geq 1$. To this end the author computes the inner product of $P_m(z, s, \Gamma)$ with a certain series $E_b(z, s, \Gamma)$ which is closely related with the Eisenstein series. Since Φ_m comes up in the constant term of P_m, this inner product can be expressed in terms of Φ_m.

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