Meise, Reinhold; Taylor, B. Alan; Vogt, Dietmar
Continuous linear right inverses for partial differential operators of order 2 and fundamental solutions in half spaces. (English) [Zbl 0876.35023]

At the beginning of the fifties L. Schwartz formulated the problem to characterize those linear partial differential operators \( P(D) \) that admit a (continuous linear) right inverse on the Fréchet space \( C^\infty(\Omega) \), \( \Omega \subset \mathbb{R}^n \) is open. This problem was solved by the authors in an earlier paper. In the present paper, the authors give a more detailed characterization of the differential operators \( P(D) \) of order 2 that admit a right inverse on \( C^\infty(\mathbb{R}^n) \).

The first step is the reduction of general quadratic polynomials to certain normal forms. A necessary and sufficient condition is proved for the operator \( P(D) \), which corresponds to a normal form, to have a right inverse on \( C^\infty(\mathbb{R}^n) \).

Among other results the following equivalence result is proved: For each non-constant polynomial \( P \in \mathbb{C}[z_1, \ldots, z_n] \) of degree 2 the following assertions are equivalent:
1. \( \mathbb{R}^n \) is \( P \)-convex with bounds, this means, \( P(D) \) has a right inverse on \( C^\infty(\mathbb{R}^n) \) if and only if \( P(D) \) has a right inverse on \( D'(\mathbb{R}^n) \);
2. there exists a basis \( \{N_1, \ldots, N_n\} \) of \( \mathbb{R}^n \) and fundamental solutions \( E_{1}^\pm, \ldots, E_{n}^\pm \) for \( P(D) \) satisfying \( \text{supp } E_{j}^\pm \subset H_\pm(N_j) \) for \( 1 \leq j \leq n \), \( H_\pm(N) := \{x \in \mathbb{R}^n : \pm(x, N) > 0\} \);
3. there exists a basis \( \{N_1, \ldots, N_n\} \) of \( \mathbb{R}^n \) for which \( H_\pm(N_j) \) is \( P \)-convex with bounds for all \( j \);
4. there exists an open half space \( H \) which is \( P \)-convex with bounds;
5. there exists a bounded convex open set \( \Omega \) in \( \mathbb{R}^n \) which is \( P \)-convex with bounds.

Finally, an example shows that this result cannot be generalized to operators of third order.

Reviewer: M. Reissig (Freiberg)

MSC:
35E20 General theory of PDEs and systems of PDEs with constant coefficients
35A08 Fundamental solutions to PDEs
35E05 Fundamental solutions to PDEs and systems of PDEs with constant coefficients
47F05 General theory of partial differential operators

Keywords:
right inverse on \( E(\mathbb{R}^n) \); fundamental solutions in half spaces; Phragmén-Lindelöf conditions

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References:


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