

Steel, John R.

The core model iterability problem. (English) Zbl 0864.03035
Lecture Notes in Logic. 8. Berlin: Springer. v, 112 p. (1996).

This monograph gives a method for constructing canonical inner models of the form $L[\mathbf{E}]$ where \mathbf{E} is a coherent sequence of extenders. These are known as core models. The goal is to produce core models with Woodin cardinals. The reader is assumed to be familiar with the fine structure theory for core models with Woodin cardinals as developed by *W. J. Mitchell* and *J. R. Steel* [Fine structure and iteration trees (Lect. Notes Logic 3) (1994; [Zbl 0805.03042](#))]. In this earlier monograph, extenders over V were used to show that the inner model $L[\mathbf{E}]$ is sufficiently iterable. In the monograph under review the extenders in \mathbf{E} are not required to be full extenders over V , and yet $L[\mathbf{E}]$ is still shown to be iterable.

The desired model K is obtained in two stages. First K^c is constructed. It has extenders which guarantee iterability. K is then a Skolem hull of K^c . The author also provides an inductive definition of K of simplest possible logical form. In order to show that K^c and K are “large enough”, something like the existence of a measurable cardinal seems to be needed.

Several applications are provided. Amongst them is the following positive answer to a conjecture of A. S. Kechris. Theorem. Assume for all $x \in \omega^\omega$ that $x^\#$ exists and $\Sigma_3^1(x)$ has the separation property. Then there is a transitive set M such that $M \models \text{ZFC} + \text{“There is a Woodin cardinal”}$.

The final sections of the monograph concern embeddings of K and a general iterability theorem.

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MSC:

- [03E45](#) Inner models, including constructibility, ordinal definability, and core models
- [03E55](#) Large cardinals
- [03-02](#) Research exposition (monographs, survey articles) pertaining to mathematical logic and foundations
- [03E10](#) Ordinal and cardinal numbers
- [03E15](#) Descriptive set theory

Cited in **5** Reviews
Cited in **68** Documents

Keywords:

[inner models](#); [extenders](#); [core models](#); [Woodin cardinals](#); [fine structure theory](#); [iterability](#); [Skolem hull](#)