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Measurable coupling of transition probability and probability metrics. (Chinese)

Zbl 0861.60007
Chin. Ann. Math., Ser. A 16, No. 6, 769-775 (1995).

This paper studies Markovian coupling for a given transition function $P(x,dy)$ on a Polish space $(E,\rho,\mathcal{E})$, where ρ is a metric on E. Roughly speaking, the author proves that if the family $\mathcal{M}:=\{P(x,\cdot):x\in E\}$ is tight and uniformly integrable in the sense that

$$\lim_{N\to\infty}\sup_{P\in\mathcal{M}}\int_{\{x:\rho(x_0,x)\geq N\}}\rho(x_0,x)P(dx)=0$$

for a fixed point $x_0\in E$, then for every $\varepsilon>0$, there exists a coupled transition probability $P(x_1,x_2,dy_1,dy_2)$ such that

$$\int\rho(y_1,y_2)P(x_1,x_2,dy_1,dy_2)\leq W(P(x_1,\cdot),P(x_2,\cdot))+\varepsilon$$

for all $x_1,x_2\in E$, where $W(P_1,P_2)$ is the Wasserstein distance of P_1 and P_2. Originally, the problem comes from the well-known Dobrushin-Shlosman uniqueness theorem for random fields. In the original proof, the measurability of $P(x_1,x_2,dy_1,dy_2)$ in (x_1,x_2) was missed. See also the reviewer's book "From Markov chains to non-equilibrium particle systems" (1992; Zbl 0753.60055), Theorem 10.9 and §10.8. Very recently, in a forthcoming paper [Acta Math. Sin.], the author improves the above result by removing ε and replacing the inequality by equality. Thus, the author finally establishes the existence theorem of ρ-optimal Markovian coupling for time-discrete Markov processes. Refer to the reviewer's paper [Acta Math. Sin., New Ser. 10, No. 3, 260-275 (1994; Zbl 0813.60068)] for further background of the study on optimal couplings.

Reviewer: Chen Mu-fa (Beijing)

MSC:

60B05 Probability measures on topological spaces
60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:

coupling; measurability; Dobrushin-Shlosman uniqueness theorem