Apter, Arthur W.

AD and patterns of singular cardinals below Θ. (English) Zbl 0855.03029

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Let Θ denote the least ordinal such that there is no function onto it which has the real line as domain. Θ is larger than ω, as the function which assigns to a real number x the absolute value of the greatest integer below x shows. Recently Steel proved that the Axiom of Determinacy implies: $\mathbf{L}[\mathbb{R}]$ satisfies that an ordinal number $\kappa < \Theta$ is an uncountable regular cardinal number if and only if it is a measurable cardinal number. This statement would be vacuously true if $\mathbf{L}[\mathbb{R}]$ were to satisfy that there are no uncountable regular cardinals below Θ. The Axiom of Determinacy also implies that $\mathbf{L}[\mathbb{R}]$ satisfies $\Theta = \aleph_\Theta$, i.e., $\mathbf{L}[\mathbb{R}]$ "thinks" there are many cardinals below Θ, and $\mathbf{L}[\mathbb{R}]$ satisfies that $\aleph_1$ and $\aleph_2$ are regular uncountable cardinal numbers.

In this paper the author gives a relative consistency result regarding the behaviour of the cofinality function below Θ; the consistency result is relative to the consistency of the Axiom of Determinacy, and uses Steel's result.

In particular, the author shows that if we assume the consistency of the Axiom of Determinacy, and if we start with ground model $\mathbf{L}[\mathbb{R}]$, and if A and B are two disjoint subsets of Θ which is a partition of the set $\{\alpha < \Theta : \aleph_\alpha \text{ regular}\}$, then an inner model $\mathbf{N}$ of a generic extension of $\mathbf{L}[\mathbb{R}]$ can be found such that $\mathbf{N}$ and the ground model have the same cardinals and have the same value of Θ, and in $\mathbf{N}$ the cofinality function behaves as follows on $\aleph_\alpha$ for $\alpha < \Theta$ (here, cof denotes cofinality in the ground model and $\mathrm{cof}^{\mathbf{N}}$ denotes cofinality in $\mathbf{N}$):

$$\mathrm{cof}^{\mathbf{N}}(\aleph_\alpha) = \begin{cases} \aleph_0 & \text{if } \alpha \in A \\ \aleph_0 & \text{if } \alpha \notin A \cup B \text{ and } \mathrm{cof}(\aleph_\alpha) = \aleph_\beta \text{ for some } \beta \in A \\ \aleph_\alpha & \text{if } \alpha \in B \\ \mathrm{cof}(\aleph_\alpha) & \text{if } \alpha \notin A \cup B \text{ and } \mathrm{cof}(\aleph_\alpha) = \aleph_\beta \text{ for some } \beta \in B. \end{cases}.$$

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03E35 Consistency and independence results

03E60 Determinacy principles

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forcing; axiom of determinacy; relative consistency result; cofinality function; inner model

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References:

[1] Proceedings of symposia in pure mathematics pp 271– (1985)
[2] Cabal seminar '77–'79 (1981) · Zbl 0456.00005
[3] Lecture notes in mathematics pp 99– (1978)
[4] Set theory (1978)
[5] Transactions of the American Mathematical Society 290 pp 41– (1985)
[6] DOI: 10.1007/BF02760939 · Zbl 0439.03036 · doi:10.1007/BF02760939
[7] DOI: 10.1007/BF01269948 · Zbl 0755.03028 · doi:10.1007/BF01269948
[8] DOI: 10.1007/BF02772571 · Zbl 0661.03040 · doi:10.1007/BF02772571
[9] DOI: 10.1016/0003-4843(78)90031-1 · Zbl 0376.02055 · doi:10.1016/0003-4843(78)90031-1
[10] DOI: 10.1016/0003-4843(71)90009-X · Zbl 0274.02034 · doi:10.1016/0003-4843(71)90009-X
[11] Descriptive set theory (1980) · Zbl 0433.03025

[12] DOI: 10.1090/S0894-0347-1989-0955605-X· doi:10.1090/S0894-0347-1989-0955605-X

[13] DOI: 10.1073/pnas.85.18.6582 · Zbl 0656.03036 · doi:10.1073/pnas.85.18.6582

[14] DOI: 10.1007/BF02771612 · Zbl 0289.02044 · doi:10.1007/BF02771612

[15] Lecture notes in mathematics (1977)

[16] DOI: 10.1016/S0003-4843(77)80002-8 · Zbl 0378.02032 · doi:10.1016/S0003-4843(77)80002-8

[17] Cabal seminar '77–' 79 pp 75– (1981) · Zbl 0456.00005

[18] The axiom of determinacy implies dependent choices in L[\(\mathbb{R}\)] 49 pp 255– (1984) · Zbl 0584.03037