Montagna, Franco; **Simi, Giulia**; **Sorbi, Andrea**
Logic and probabilistic systems. (English) Zbl 0854.03020
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The notion of a probabilistic system, based on the two predicates of the form $R(\varphi, q)$ = "the probability value of φ is eventually $\leq q$" and $S(\varphi, q)$ = "the probability value of φ is eventually $\geq q$", can be defined as a pair $\langle R, S\rangle$ of relations between sentences and rational numbers satisfying certain axioms which formalize these predicates. The authors connect trial and error probabilistic functions with probabilistic systems and show that, although every trial and error probabilistic function generates a probabilistic system, there exist probabilistic systems that are not generated by any trial and error probabilistic function. It is also shown that, for a given probabilistic system $\langle R, S\rangle$, if $\pi_{R,S}$ is defined on all sentences, then $\pi_{R,S}$ is the limit of a trial and error probabilistic function, where the partial function $\pi_{R,S}$ satisfies properties of measure and can be presented by means of the inner and the outer probability of $\varphi : \underline{\pi}_{R,S}(\varphi) = \sup\{q \mid S(\varphi, q)\}$ and $\overline{\pi}_{R,S}(\varphi) = \inf\{q \mid R(\varphi, q)\}$, as their common value, when they coincide. It is proved that the relations R and S corresponding on a trial and error probabilistic function can be Σ_2-complete and that the set of sentences whose limit probability value is 1 is always Π_3 and can be Π_3-complete, but for any probabilistic system $\langle R, S\rangle$, the set of sentences φ for which $\pi_{R,S}(\varphi) = 1$ can not contain all the Π_2 true sentences. One of the interesting results of the paper is that, under reasonable conditions, no probabilistic system can be improved, where improving a probabilistic system $\langle R, S\rangle$ would mean to find a probabilistic system $\langle R', S'\rangle$ such that, for any true sentence φ, $\underline{\pi}_{R,S}(\varphi) \leq \underline{\pi}_{R',S'}(\varphi)$ and $\overline{\pi}_{R,S}(\varphi) \leq \overline{\pi}_{R',S'}(\varphi)$, but, for some true φ, $\pi_{R,S}(\varphi) < \pi_{R',S'}(\varphi)$. The paper is self-contained so that no special background in probability or measure theory is needed.

Reviewer: B.Boričić (Heraklion)

MSC:

03B48 Probability and inductive logic
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