Skopenkov, A. B.

Geometric proof of Neuwirth's theorem on the construction of 3-manifolds from 2-dimensional polyhedra. (English. Russian original) Zbl 0853.57003

Math. Notes 56, No. 2, 827-829 (1994); translation from Mat. Zametki 56, No. 2, 94-98 (1994); Errata ibid. 59, No. 6, 914 (1996).

J. R. Stallings [Fundam. Math. 51, 191-194 (1962; Zbl 0121.40006)] proved that there is no algorithm for checking if a given group is the fundamental group of some 3-dimensional manifold. *L. Neuwirth* [Proc. Camb. Philos. Soc. 64, 603-613 (1968; Zbl 0162.27603)] described an algorithm for determining if a 2-dimensional polyhedron, given as a CW-complex with one vertex, can be thickened to a 3-dimensional manifold. We describe an algorithm for determining if any given 2-dimensional polyhedron can be thickened to some (or some orientable) 3-dimensional manifold. An algorithm for arbitrary CW-complexes is constructed similarly. This problem cannot be reduced to a problem for a complex with one vertex by a contraction of the maximal tree. Indeed, a contraction of an edge of a non-thickenable polyhedron can produce a thickenable polyhedron (for example, if N is the Möbius band with a disk glued to it along its middle line and $I \subset N$ is the projection of the Möbius band onto the middle line, then N/I is thickenable, whereas N is not).

In this article we use a simpler and more geometric method of proof than the one used by Neuwirth. Our method is based on reducing the property of being thickenable to embeddings of graphs into a sphere. Corollaries of our approach are *P. Wright*'s theorem on the thickenability of fake surfaces [Topology 16, 435-439 (1977; Zbl 0378.57008)] and verification of the approximability of a mapping of the graph into the plane by embeddings. This solves a problem set by E. V. Shchepin. See [*K. Sieklucki*, Fundam. Math. 65, 325-343 (1969; Zbl 0197.49201)] for related questions on embeddings of mappings.

MSC:

57M20 Two-dimensional complexes (manifolds) (MSC2010)
57N10 Topology of general 3-manifolds (MSC2010)

Keywords:

2-dimensional polyhedron; 3-dimensional manifold; algorithm; polyhedron; embeddings; graphs

Full Text: DOI

References:

[1] J. Stallings, "On the recursiveness of sets of presentation of 3-manifold groups," Fund. Math.,51, 191–194 (1962–63).

[2] L. Neuwirth, "An algorithm for the construction of 3-manifolds from 2-complexes," Proc. Camb. Phil. Soc. Math. Phys. Sci.,64, 603–613 (1968). · Zbl 0162.27603 · doi:10.1017/S0305004100043279

[3] P. Wright, "Covering of 2-dimensional polyhedra by 3-manifold spines," Topology,16, 435–439 (1977). · Zbl 0378.57008 · doi:10.1016/0040-9383(77)90048-9

[4] K. Sieklucki, "Realization of mappings," Fund. Math.,65, No. 3, 325–343 (1969). · Zbl 0197.49201

[5] R. Rourke and S. Sanderson, "Introduction to piecewise-linear topology," Ergerbn. Math.,69, Springer-Verlag, Berlin, New York (1972). · Zbl 0254.57010

[6] R. N. Bing, "The geometric topology of 3-manifolds," AMS Colloq. Publ.,40 (1983). · Zbl 0535.57001