Dumas, François; Jordan, David A.

The 2×2 quantum matrix Weyl algebra. (English) Zbl 0851.16025

Commun. Algebra 24, No. 4, 1409-1434 (1996).

The algebras of differentials on quantum affine spaces introduced by *G. Maltsiniotis* [Calcul différentiel quantique, Groupe de travail, Université de Paris VII (1992)] have been studied from the point of view of noncommutative ring theory in a number of papers – e.g., *Akhavizadegan* and the second author [Prime ideals of quantized Weyl algebras (Glasg. Math. J., to appear)]; *J. Alev* and the first author [J. Algebra 170, No. 1, 229-265 (1994; Zbl 0820.17015)]; *G. Cauchon* [J. Algebra 180, No. 2, 530-545 (1996; Zbl 0849.16028)]; *T. H. Lenagan* and the reviewer [J. Pure Appl. Algebra 111, 123-142 (1996)]; the second author [J. Algebra 174, No. 1, 267-281 (1995; Zbl 0833.16025)]; and *L. Rigal* [Beitr. Algebra Geom. 37, No. 1, 119-148 (1996)]. Here the authors consider an algebra $W_{p,q}$ of differentials on two-parameter 2×2 quantum matrices defined by *G. Maltsiniotis* [in Commun. Math. Phys. 151, No. 2, 275-302 (1993; Zbl 0783.17007)], and investigate its similarities with the quantum Weyl algebras $A_n^{\overline{q},\Lambda}$ studied earlier. Similarities: $W_{p,q}$ has a simple localization of Krull and global dimension 4 obtained by inverting a finite set of normal elements, and this localization is isomorphic to a corresponding localization of an $A_4^{\overline{q},\Lambda}$ for suitable choices of parameters $\overline{q}, \Lambda$. Dissimilarity: $W_{p,q}$ has 3 height 1 primes, rather than 4 as in any $A_4^{\overline{q},\Lambda}$. In particular, $W_{p,q}$ is not isomorphic to any $A_4^{\overline{q},\Lambda}$. These properties of $W_{p,q}$ are derived by presenting the algebra as an iterated skew polynomial ring in such a way that a generalization of the techniques developed by the second author [ibid.] can be applied.

Reviewer: K.R.Goodearl (Santa Barbara)

MSC:

16S36 Ordinary and skew polynomial rings and semigroup rings
17B37 Quantum groups (quantized enveloping algebras) and related deformations
16D25 Ideals in associative algebras
16P40 Noetherian rings and modules (associative rings and algebras)
16P50 Localization and associative Noetherian rings
16P60 Chain conditions on annihilators and summands: Goldie-type conditions

Keywords:

algebras of differentials on quantum affine spaces; 2×2 quantum matrices; quantum Weyl algebras; simple localizations; Krull and global dimensions; normal elements; height 1 primes; iterated skew polynomial rings

Full Text: DOI

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