Hájek, Petr; **Godo, Lluis**; **Esteva, Francesc**

A complete many-valued logic with product-conjunction. (English) Zbl 0848.03005

Arch. Math. Logic 35, No. 3, 191-208 (1996).

Studies in the field of fuzzy sets related to triangular norms have shown that the infinitely many-valued logic over the real unit interval with 1 as the only designated truth degree and a product-based conjunction connective has not been discussed much till now, despite some representation theorems of t-norms which refer to the product as a kind of basic t-norm.

Choosing the product conjunction, a corresponding implication connective via residuation (i.e., both of them as an adjoint pair), and defining negation formally from implication and the truth degree constant 0 like in intuitionistic logic, the authors constitute the system they investigate.

The completeness proof is given via algebraic studies essentially like the corresponding proof for Łukasiewicz's infinite-valued logic, but now introducing and investigating product algebras for this product logic instead of the MV-algebras for the Łukasiewicz case.

Reviewer: S.Gottwald (Leipzig)

MSC:

03B50 Many-valued logic
03B52 Fuzzy logic; logic of vagueness
03G25 Other algebras related to logic

Keywords:

axiomatizing product conjunction; infinite-valued logic; product algebras; product logic

Full Text: DOI

References:

[1] Alsina, C., Trillas, E., Valverde L.: On some logical connectives for fuzzy set theory. J. Math. Anal. Appl.93, 15–26 (1983) · Zbl 0522.03012 · doi:10.1016/0022-247X(83)90216-0

[2] Balbes, R., Dwinger, P.: Distributive lattices. Missouri: Univ. Missouri Press 1974 · Zbl 0321.06012

[3] Birkhoff, G.: Lattice theory, vol. 25. New York: 1948, Amer. Math. Soc. Colloquium Publ. · Zbl 0033.10103

[4] Bouchon, B.: Fuzzy inferences and conditional possibility distributions Fuzzy Sets Syst.23, 33–41 (1978) · Zbl 0633.68100 · doi:10.1016/0165-0114(87)90098-4

[5] Fuchs, L.: Partially ordered algebraical systems. New York: Pergamon Press 1963 · Zbl 0137.02001

[6] Gödel, K.: Zum intuitionistischen Aussagenkalkül. Anz. Akad. Wissensch. Wien, Math.-naturwissensch. Klasse69, 65–66 (1932). Erg. math. Kolloqu.4, 40 (1933) · Zbl 58.1001.03

[7] Gottwald, S.: Mehrwertige Logik. Berlin: Akademie-Verlag 1988

[8] Grätzer, G.: Universal Algebra. Berlin Heidelberg New York: Springer 1979 · Zbl 0412.08001

[9] Gurevich, Y., Kokorin, A.I.: Universal equivalence of ordered Abelian groups (in Russian). Algebra i Logika2.1, 37–39 (1963)

[10] Hájek, P.: Fuzzy logic and arithmetical hierarchy. Fuzzy Sets Syst.73, 359–363 (1995) · Zbl 0857.03011 · doi:10.1016/0165-0114(94)00299-M

[11] Hájek, P.: Fuzzy logic and arithmetical hierarchy, vol. II. Submitted · Zbl 0857.03011

[12] Hájek, P., Havránek, T., Jiroušek, R.: Uncertain information processing in expert systems. CRC Press 1992

[13] Hájek, P., Valdés, J.J.: Algebraic foundations of uncertainty processing in rule-based expert systems I. Comp. Artif. Intell.9, 325–334 (1990)

[14] Höhle, U.: Commutative residuated monoids. In: Höhle, U., Klement, P., (eds) Non-classical logics and their applications to fuzzy subsets (A handbook of the mathematical foundations of the fuzzy set theory). Dordrecht. Kluwer 1995

[15] Ling, C.H.: Representation of associative functions Publ. Math. Debrecen12, 182–212 (1965)

[16] Łukasiewicz, J.: Selected works. Amsterdam: North-Holland 1970 · Zbl 0212.00902

[17] Novák, V.: On the syntactico-semantical completeness of first-order fuzzy logic I, II Kybernetika26, 47–26, 134–152 (1990) · Zbl 0705.03009

[18] Paris, J.B.: The uncertain reasoner's companion – a mathematical perspective. Cambridge: Cambridge University Press 1994 · Zbl 0838.68104

[19] Pavelka, J.: On fuzzy logic I, II, III. Z. Math. Logik Grundl. Math.25, 45–52, 119–134, 447–464 (1979) · Zbl 0435.03020 · doi:10.1002/malq.19790250304

[20] Rose, A., Rosser, J.B.: Fragments of many-valued statement calculi. Trans. A.M.S.87, 1–53 (1958) · Zbl 0085.24303 · doi:10.1090/S0002-9947-1958-0094299-1

[21] Scarpelini, B.: Die Nichtaxiomatisierbarkeit des unendlichwertigen Prädikatenkalküls von łukasiewicz. J. Symb. Log.27, 159–170 (1962) · Zbl 0112.24503 · doi:10.2307/2964111

[22] Schweizer, B., Sklar, A.: Associative functions and abstract semi-groups. Publ. Math. Debrecen10, 69–81 (1963) · Zbl 0119.14001

[23] Schweizer, B., Sklar, A.: Probabilistic metric spaces. Amsterdam: North Holland 1983 · Zbl 0546.60010

[24] Takeuti, G., Titani S.: Fuzzy Logic and fuzzy set theory. Anal. Math. Logic32, 1–32 (1992) · Zbl 0786.03039

[25] Zadeh, L.: Fuzzy logic. IEEE Comput.1, 83 (1988) · Zbl 0634.03020 · doi:10.1109/2.53