

**Coleman, Thomas F.; Li, Yuying**

**On the convergence of interior-reflective Newton methods for nonlinear minimization subject to bounds.** (English) [Zbl 0842.90106](#)

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The authors analyze a new affine-scaling interior-point method for the minimization of a nonlinear function  $f$  subject to simple bounds on the variables. Unlike the Dikin-Karmarker scaling matrix, the scaling matrix  $D(x)$  used in this paper is generated using the distance of the iterates to the bounds and the direction of the gradient. Components of the variables are only scaled by the square root of their distance to the bound if the corresponding component of the negative gradient points to that bound. Using this scaling matrix  $D(x)$ , the first order necessary conditions can be written as  $D(x)\nabla f(x) = 0$ . Search directions are computed using a modified Newton method applied to  $D(x)\nabla f(x) = 0$ . To improve global convergence, in particular to avoid short steps generated because some variables may approach the wrong bounds, reflections of the Newton steps at the bounds are used. Global convergence and local  $q$ -quadratic convergence of this method is proven.

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**MSC:**

[90C30](#) Nonlinear programming

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[affine-scaling interior-point method](#); [global convergence](#); [local  \$q\$ -quadratic convergence](#)

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